STEMscopes™
MATH

Take & Teach

Accelerate LEARNING
THE LEADER in STEM EDUCATION

What's Inside This Sample Lesson?

- A fully guided **Explore activity** written to meet rigorous state and national standards
- **Teacher Edition** pages, **Student Workbook** pages, and **other helpful resources** to fully experience a STEMscopes Math Explore activity
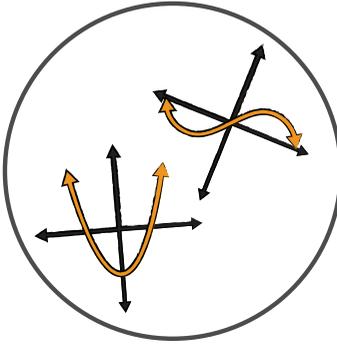
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Algebra 1, Properties of Functions - Explore 2

Go Online!

Explore the digital resources for this lesson.

Properties of Functions
Scope Introduction

SCOPE SUMMARY

In this scope, students will learn through exploration how to determine whether a relation is a function; identify and explain that each function has a domain and a corresponding range; use function notation; determine a reasonable domain and range for real-world situations; evaluate functions when the input is given as a numeric value, an algebraic expression, or a graph; interpret functions and match or connect them to real-world contexts; calculate the average rate of change of a function (presented symbolically or as a table) over a specified interval; use the average rate of change and relate it to determining the slope; and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval based on the context.

Student Expectations

Create equations that describe numbers or relationships.

- Create equations and inequalities in one variable, including ones with absolute value, and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
- Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

Solve equations and inequalities in one variable.

- Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

VERTICAL ALIGNMENT

Background Knowledge

In previous grade levels, students created and solved inequalities in one variable, created and solved linear equations, learned and applied the properties of operations, and reasoned with inequalities. Students also recognized one important new consideration in solving inequalities: Multiplying or dividing both sides of an inequality by a negative number reverses the order of the comparison it represents.

Future Expectations

In the future, students will use compound inequalities to make sense of absolute value inequalities. This material is also a common prerequisite for finding inverse functions, including the inverse relationship of exponential and logarithmic functions. In Algebra II, students will solve rational and radical inequalities.

ENGAGE ACTIVITIES

Accessing Prior Knowledge

To assess students' prior knowledge, the teacher should assess whether students can match numbered cards with lettered cards posted around the room. Students are expected to understand that a function is a rule that assigns to each input exactly one output and the graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

If your students are struggling with previously taught concepts, use the Foundation Builder activity in this scope to reinforce ideas presented in the APK.

Hook

The Hook is introduced at the start of the scope and completed after the exploration activities. At the beginning of the Hook, students will discuss their noticings, wonderings, and where they see the math in this scenario. Students will relate function notation to a real-world situation by creating a model about the amount of charge a cell phone has as time passes.

Home	Engage	Explore	Explain	Elaborate	Evaluate	Intervention	Acceleration

EXPLORE ACTIVITIES

Explore 1

Explore 1 — Relations and Functions

In the first exploration, students will work in small groups to analyze the properties of functions and non-functions through various types of algebraic representations in order to develop the definition of a function. They will also be able to decipher between continuous and discrete functions. Groups will be working collaboratively to solve a real-world problem as an employee of a Widget factory and asked to analyze the inputs and outputs of each widget machine to determine which two are functioning machines and which machine is not functioning properly. After students complete the activity, they share their learning and conclude with an Exit Ticket.

Explore 2

Explore 2 — Evaluating Functions

In the second exploration, students will evaluate linear and quadratic functions in function notation and interpret function notation within various contexts. Small groups will work together to analyze data about tracking the distance traveled by one nest of leatherback sea turtles over time. Groups will need to interpret the data that is displayed on a graph and use the information to complete the report on leatherback sea turtle migration. After students complete the activity, they share their learning and conclude with an Exit Ticket.

JUMP IN HERE

Explore 3

Explore 3 — Average Rate of Change

In small groups, students will explore how to calculate and interpret the average rates of change over specified intervals. Students will connect the rates of change across representations and make generalizations about the behaviors of functions over time. The real-world scenario that groups are solving is about analyzing food sales from the previous year and identifying average changes in sales for specific time periods for a Student Celebration Day for the school. The groups will determine the chosen foods that have proven popularity. After students complete the activity, they share their learning and conclude with an Exit Ticket.

Explore 4

Domain and Range

In the final exploration, groups will determine the domains and ranges of functions and will write those values using appropriate formats. Students will also determine domain and range as they relate to the context of a problem and be able to differentiate between discrete and continuous situations. The groups will work collaboratively to track ticket sales for 7 days prior to a student event to create a report about the anticipated attendance at the event, provide enough food and prizes, and track attendance throughout the 4-hour event. After students complete the activity, they share their learning and conclude with an Exit Ticket. After students complete the activity, they share their learning, complete an Exit Ticket, and revisit the Hook to solve.
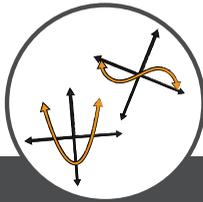
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Properties of Functions
Explore 2 – Evaluating Functions

ACTIVITY PREPARATION

Students will evaluate linear and quadratic functions in function notation and interpret function notation within various contexts.

Standards of Mathematical Practice

- **MP.3 Construct viable arguments and critique the reasoning of others:** Students will analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments.
- **MP.4 Model with mathematics:** Students will apply the math they know to solve problems in everyday life.
- **MP.8 Look for and express regularity in repeated reasoning:** Students will understand the broader application of patterns and see the structure in similar situations.

Materials

Printed
- 1 Student Journal (per student)
- 1 Exit Ticket (per 2 students)

Preparation

- Separate the class into groups of 2 or 3 students.
- Print a Student Journal for each student.
- Print an Exit Ticket for every 2 students. Cut apart the half-page Exit Tickets so each student has one.

PROCEDURE AND FACILITATION POINTS

FACILITATION TIP

*Before reading the scenario, ask the class
1) What is your favorite zoo animal?;
2) What information do you know about your favorite animal?; 3) How can you find out more information about your favorite animal's habits?*

STEMscopes Tip

The Mathematical Modeling Task is a performance-based assessment found in the Evaluate section. It assesses student mastery of content by requiring them to apply math skills to real-world contexts. The task can be completed independently, in pairs, or small groups and can be printed or assigned digitally. A rubric is provided to give constructive feedback to students.

FACILITATION TIP

After students answer the question, ask them what they can conclude about the slope of the graph from f(5) to f(10). If students do not mention constant slope in their answer, review rate of change and tie it to slope. Then, review how slope is constant for linear functions with the class.

Part I

1. Read the following scenario to the class: *You have recently obtained a position as an assistant zoologist at your local zoo. The head zoologist, Dr. Angola, has assigned you to your first research project, in which you are to find out more information about the migration behavior of leatherback sea turtles. Dr. Angola's team has collected data to track the distance traveled by one nest of leatherback sea turtles over time. The data is displayed on a graph, but you need to interpret the graph and use the information to complete the report on leatherback sea turtle migration.*

2. Give a Student Journal to each student.

3. Explain to students that they will work with their groups to analyze the information regarding turtle migration on the graph. Students will devise a strategy for completing the missing pieces of data in the report on leatherback sea turtle migration and answer the questions that follow in Part I on their Student Journals.

4. As students collaborate, monitor their work and use the following guiding questions to assess student understanding:

   a. **DOK-1** How can you use $f(5) = 50$ to help you fill in the blank for $f(\_\_\_) = 80$? In row *A*, the 5 represents the days it took to travel 50 miles. So in row *B*, 8 must go in the blank because it took 8 days to travel 80 miles. The number in the parentheses must represent the days.

   b. **DOK-1** What does the coordinate (8, 80) represent on the graph? The coordinate (8, 80) means that the turtles traveled 80 miles in 8 days.

5. Allow students enough time to complete Part I and answer the questions that follow.

6. After completing Part I, invite the class to a Math Chat to share their observations and learning thus far.

| Home | Engage | Explore | Explain | Elaborate | Evaluate | Intervention | Acceleration |

Math Chat

- **DOK-1** Normally, a number next to parentheses represents multiplication. Is that the case here? Explain. *Here, the number inside the parentheses is telling us the input, or x-coordinate.*
- **DOK-2** What is the difference between $f(20)$ and $f(x) = 20$? *$f(20)$ is asking us to find the distance traveled after 20 days, while $f(x) = 20$ is asking us to find the number of days when the turtles traveled 20 miles.*
- **DOK-1** When considering the function in question 1, $f(x) = 8x$, what is the dependent variable (output), and what is the independent variable (input)? *The dependent variable is $f(x)$, which stands for the distance traveled. This is the value of the output. The independent variable is x, which stands for the number of days. This represents the value of the input.*
- **DOK-2** How is $f(x) = 8x$ different from $y = 8x$? How are they similar? *They both represent the same pattern and can be used interchangeably.*
- Explain the following to the class: *Function notation was created by mathematicians so they had a shorter way to discuss long or complex equations. Function notation is similar to using LOL or TL;DR—they are all shorthand ways of saying something. If we were analyzing a situation that used multiple equations, we could use function notation. We might use $f(x)$, $g(x)$, or $h(x)$ to name and discuss the functions. The notation $g(x)$ represents the output of a function named g when it is evaluated at a value of x.*

Part II

1. Read the following scenario to the class: *Leatherback sea turtles are not the only sea turtle species Dr. Angola's team has been tracking. Dr. Angola was so pleased with your report on leatherbacks that she has asked you to complete the report on multispecies migration. The team has developed functions to represent the distances traveled over any amount of time for each species studied.*
2. Explain to students that they will work with their groups to evaluate functions for the given value or find the input for a given output. Students will use these calculations to complete the report on multispecies migration in Part II of their Student Journals.
3. As students collaborate, monitor their work and use the following guiding questions to assess student understanding:

   a. **DOK-1** What does $f(14)$ mean? *This means to evaluate the given function with an input of 14.*

   b. **DOK-1** Once you calculate that $f(14) = 576$, how do you determine which number represents days and which number represents miles traveled? *I know 14 is the input or independent variable, and 576 is the output or dependent variable. In this scenario, the number of days is the independent variable, and the number of miles traveled is the dependent variable.*

   c. **DOK-1** How do you evaluate a function that has more than one x (for example, $g(x) = (x + 4)(x − 2)$)? *I substitute the given input for each appearance of x and then use order of operations to simplify.*

   d. **DOK-2** If $f(14)$ represents the distance traveled by flatbacks in 14 days and $g(14)$ represents the distance traveled by greens in 14 days, what would $f(14) + g(14)$ represent? *This would represent the combined distance traveled by flatbacks and greens in 14 days.*

4. Allow students enough time to complete Part II and answer the questions that follow.
5. After the Explore activity, invite the class to a Math Chat to share their observations and learning.

FACILITATION TIP

After students answer the question, remind them what an input is and what an output is. As you remind them, tie input to independent variable and output to dependent variable. Then, ask the class which instance of "20" is an input and which one is an output.

FACILITATION TIP

Have the class look at the graph in Part I. Ask them if there is any section before Point D on the graph where f(x)=8x, and have them justify their answer. Use the graph to introduce or further discuss a steeper slope versus a shallower slope.

FACILITATION TIP

Before reading the scenario, ask the class 1) What other animals besides sea turtles do you know of that migrate?; 2) Where do they go when they migrate?; 3) What time of the year do they migrate?

STEMscopes Tip

Each scope includes a Home section accessed along the scope's menu bar. Here you will see student expectations as well as key concepts and fundamental questions. Each Home tab includes drop-down options to access the Scope Overview, Content Support, Content Unwrapped, Materials List, and Parent Letter pages.

FACILITATION TIP

Student Journal Part II, Reflect: For Question 1, watch out for students leaving out negative signs or using them incorrectly when solving for the function value. They may forget that squaring a negative value results in a positive value. They may also leave out the negative sign when simplifying the linear value.
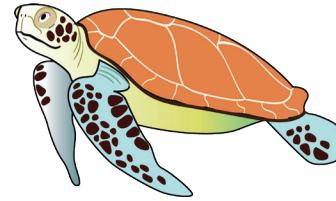
Properties of Functions

Explore 2 – Evaluating Functions

FACILITATION TIP

After students answer the question, ask them for other examples of scenarios where it would not make sense to evaluate a function using a negative number. If they are stumped, present length and volume and ask for other examples. Then, ask them for examples of scenarios where it could make sense to evaluate a function using a negative number. If they are stumped, present position and temperature and ask for other examples.

Math Chat

- **DOK-2** For flatback turtles, $f(20) > f(15)$. Explain what this means in context and what this information tells you about the graph of f(x). Since $f(20) > f(15)$, the flatback turtles have traveled farther after 20 days than after 15 days. On a graph, the output, or y value, when $x = 20$ would be higher than the y value when $x = 15$.

- **DOK-2** Is it possible to evaluate a function using a negative number? Does it make sense in this scenario? Explain. Yes, it is possible. You would substitute and simplify just as you would with a positive number. However, it does not make sense in this scenario because our inputs represent days, and the turtles cannot travel for a negative amount of days.

- **DOK-2** What are some advantages to using function notation? Function notation is more flexible than using "y = ." It is easier to keep track of different functions in the same scenario using function notation: $f(x)$, $g(x)$, $h(x)$, etc.

6. When students are done, have them complete the Exit Ticket to formatively assess their understanding of the concept.

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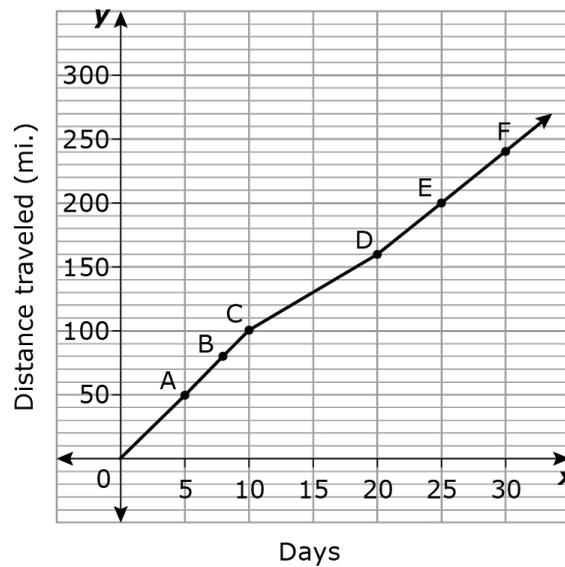_____
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Explore

Name:_____ Date:_____

Turtle Tracker

Part I
Analyze the graph below showing the data collected by Dr. Angola's team. Use the graph and the data to complete the report for Dr. Angola.

Tracked Sea Turtle Migration

Days

			Report: Leatherback Sea Turtle Migration
A	(5, 50)	$f(5) = 50$	After 5 days, the turtles had traveled 50 miles.
B	(8, 80)	$f(\_\_\_) = 80$	After _____ days, the turtles had traveled 80 miles.
C	(10, 100)	$f(10) = \_\_\_\_$	After 10 days, the turtles had traveled _____ miles.
D			After _____ days, the turtles had traveled _____ miles.
E			After _____ days, the turtles had traveled _____ miles.
F			After _____ days, the turtles had traveled _____ miles.

1

Explore

1. Dr. Angola is most interested in the section of the graph containing points D, E, and F, as she believes the turtles will continue on this trend for the next several days. Complete the function below to represent the relationship between the number of days and the distance traveled for this section of the graph.

$$f(x) = \underline{\hspace{2cm}}$$

2. Use your equation from question 1 to predict the distance traveled by the turtles after 36 days.

Part II

Evaluate the functions for the given value, or find the input for a given output. Complete the multispecies report for Dr. Angola.

Report: Multispecies Migration		
Flatbacks $f(x) = 4(x - 2)^2$	$f(14) = \underline{\hspace{1cm}}$	After _____ days, the Flatbacks had traveled _____ miles.
Archelons $a(x) = 7x$	$a(x) = 126$ $x = \underline{\hspace{1cm}}$	After _____ days, the Archelons had traveled _____ miles.
Loggerheads $l(x) = x^2 + 5x$	$l(9) = \underline{\hspace{1cm}}$	After _____ days, the Loggerheads had traveled _____ miles.
Hawksbills $h(x) = 18x$	$h(x) = 54$ $x = \underline{\hspace{1cm}}$	After _____ days, the Hawksbills had traveled _____ miles.
Greens $g(x) = (x + 4)(x - 2)$	$g(6) = \underline{\hspace{1cm}}$	After _____ days, the Greens had traveled _____ miles.
Kemp's Ridleys $k(x) = 16x + 10$	$k(x) = 170$ $x = \underline{\hspace{1cm}}$	After _____ days, the Kemp's Ridleys had traveled _____ miles.

2

Explore

Reflect

1. Consider the function $t(x) = x^2 + 8x - 4$.
   a. Find $t(-4)$.

   b. If $t(x)$ represents the distance traveled by Terrapin sea turtles over time, does $t(-4)$ make sense? Explain.

2. Dr. Angola explained that Archelons and Loggerheads often swim for 3 days and then take a break. She wants to know the total distance traveled by these two species after 3 days. Using the functions from the report on multispecies migration, evaluate $a(3) + l(3)$.

3. Why might a student think that $f(x)$ means to multiply f times x?

4. Explain what "evaluate $g(5)$" means in your own words.

5. How is solving the equation $f(x) = 10$ different from evaluating the expression $f(10)$?

3

💡 **Explore**

Name:_____ Date:_____

Evaluating Functions Exit Ticket
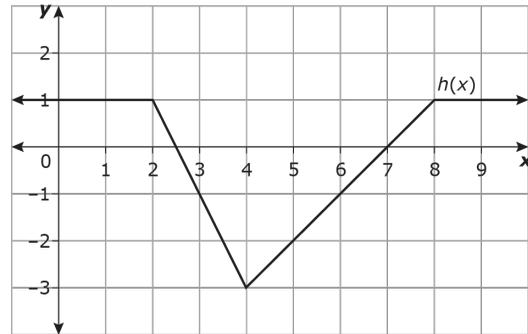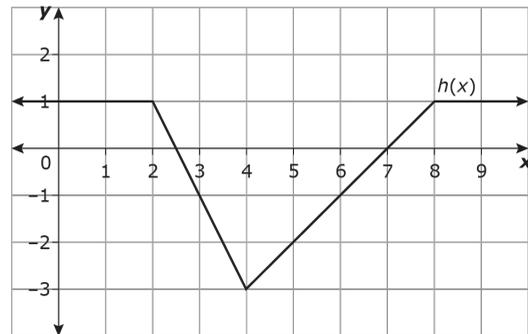
1. Use the graph to determine the following values:
   a. $h(2) =$ _____
   b. $h(7) =$ _____
   c. $h(x) = -3$, $x =$ _____

2. Given that $f(x) = 3x$ and $g(x) = x^2 - 7x + 2$, find the following values:
   a. $f(2) =$ _____
   b. $g(-1) =$ _____
   c. $f(2) + g(-1) =$ _____

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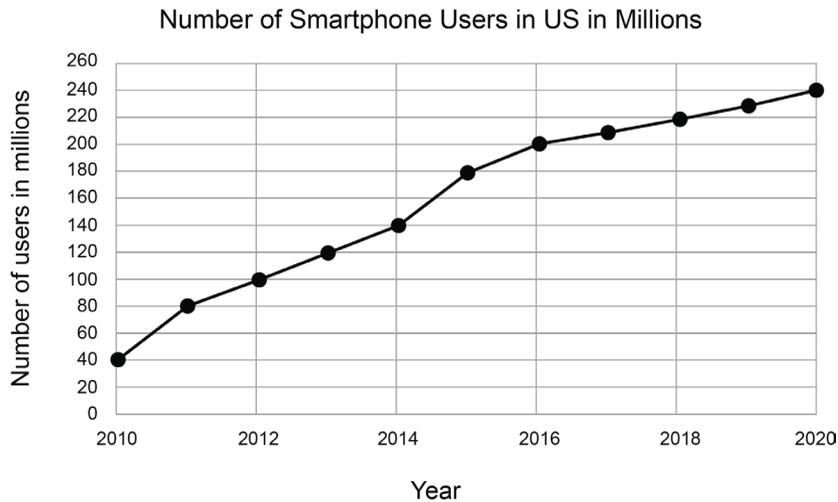
Show What You Know

Name:_____ Date:_____

Evaluating Functions

Rodrigo was writing a report on the increase in the use of cell phones. He found this graph of data on the number of smartphone users in the US.

Number of Smartphone Users in US in Millions

1. If we use $f(x)$ to describe this data as a function, what variable does x represent?

2. $f(2012) =$ _____

3. $f(2016) =$ _____

4. If $f(x) = 140$ million, then $x =$ _____.

5. Evaluate $f(2020) - f(2010)$, and describe what it represents in context.

1

Show What You Know

6. Consider the function $g(x) = 3x - 1$

   a. If $g(x) = 2$, what is x?

   b. If $g(x) = -4$, what is x?

7. Given that $a(x) = x^2 + 5x + 6$ and $b(x) = 4x - 10$, find solutions to the following equations:

   a. $a(3) =$ _____

   b. $b(3) =$ _____

   c. $a(3) - b(3) =$ _____

   d. $\dfrac{a(3)}{b(3)} =$ _____

8. Use the table on the right to answer the questions below.

   a. $g(2) =$ _____

   b. $h(\_\_) = 10$

   c. $g(1) =$ _____

   d. $h(3) - g(5) =$ _____

   e. $g(\_\_) = 2$

   f. $g(\_\_) \cdot h(4) = 40$

x	$g(x)$	$h(x)$
1	5	2
2	4	4
3	3	6
4	2	8
5	1	10

2

Properties of Functions
Part 2

Show What You Know

9. Create a graph below that models the following tide scenario. Let $x = 0$ represent midnight when the tide is at a high of 9 feet in the harbor. At 3 a.m., the tide is at its midline of 5 feet and at 6 a.m., the tide in the harbor reaches a low of 1 foot. 6 hours later the tide reaches its high of 9 feet again, and 6 hours after that, the tide is back at the low of 1 foot. If the tide continues along according to this pattern, graph the height of the water in the harbor below.

Height of tide (feet)

Time since midnight (hours)
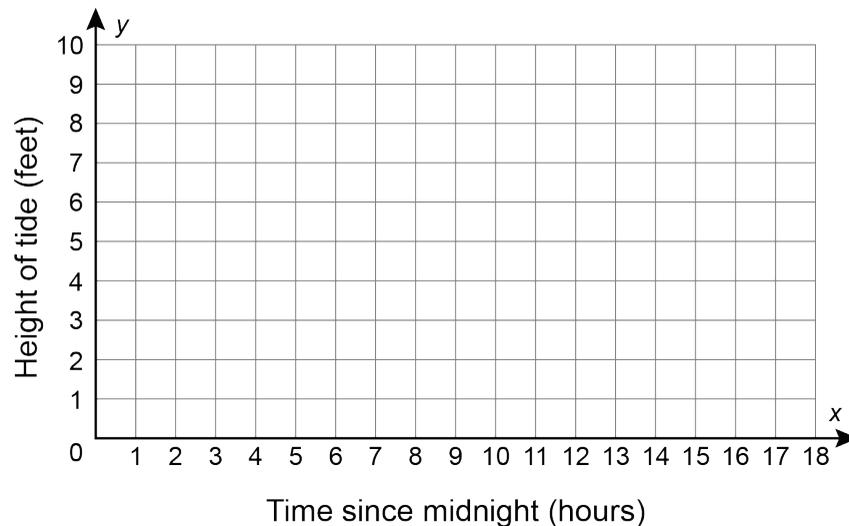
10. The graph you created is called a *trigonometric function* and it continues to oscillate or travel up and down. If the function you graphed in question nine is labeled $f(x)$, how could you determine the value of $f(36)$ based on the key features of the graph you created?

3

Notes

Notes

Notes

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