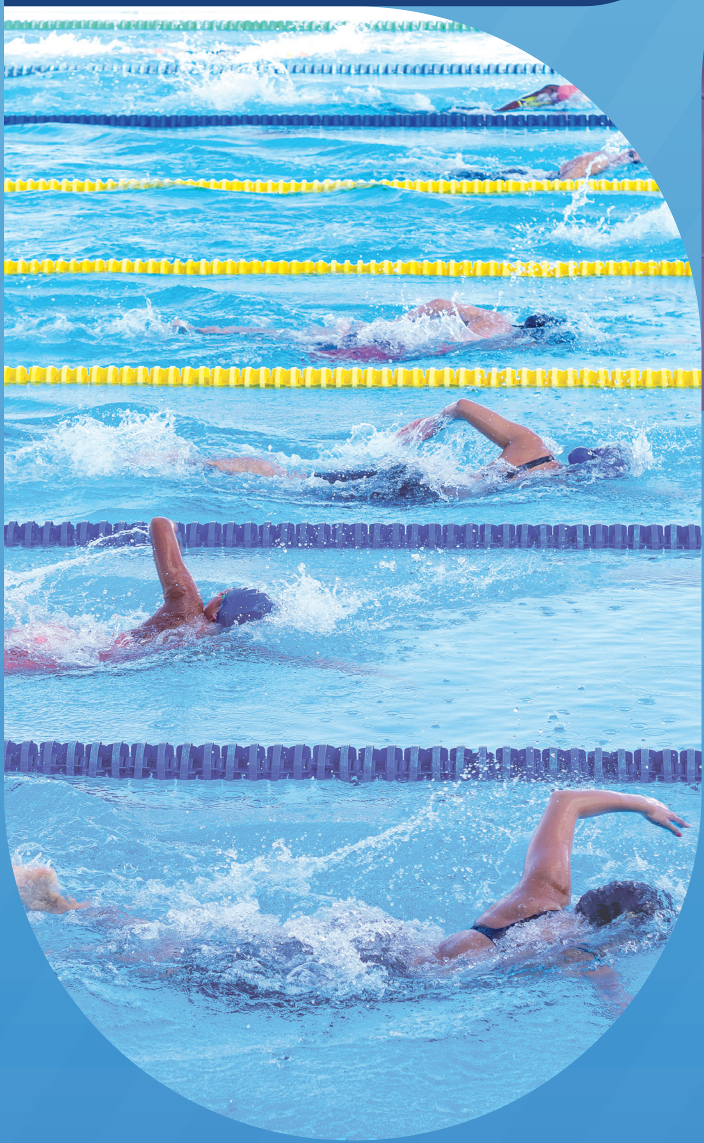
Geometry

MATH NATION
BY ACCELERATE LEARNING

Take & Teach



AccelerateLEARNING
THE LEADER in STEM EDUCATION

What’s Inside This Sample Lesson?

- A fully guided lesson written to meet rigorous state and national standards
- **Teacher Edition** pages, **Student Workbook** pages, and other **helpful resources** to fully experience a Math Nation lesson
- Warm-ups, exploration tasks, instructional routines, and teacher prompts
- Support for English learners and students with disabilities
- Integrated reflection, synthesis, and cool-down opportunities

Table of Contents

Geometry, Unit 5, Lesson 2 - Slicing Solids

Go Online!

Explore the digital resources for this lesson.

LESSON 2
SLICING SOLIDS

LEARNING GOALS

- *Generate multiple cross sections of three-dimensional figures.*
- *Identify the three-dimensional shape resulting from combining a set of cross sections.*

ALIGNMENT

Building On

- **7.G.A.3** Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Addressing

- **HSG-GMD.B.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Building Toward

- **HSG-GMD.A.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.

LESSON PREPARATION

Required Materials

- Clay
- Coins
- Any fair two-sided coin
- Cylindrical food items
- Approximately cylindrical food items that can be easily sliced. Examples include carrots or cheese sticks.
- Dental floss

Required Preparation

- Obtain several cylindrical food items to cut with a plastic knife.
- Devices are required for the digital version of the activity Slice That. If using the paper and pencil version, prepare various solids from clay or play dough, such as cubes, spheres, cones, and cylinders. Each group of 3–4 students should have access to a three-dimensional solid to analyze.
- Alternatively, you might consider getting food items from the grocery store with interesting cross sections or three-dimensional foam solids from a craft store, and plastic knives to slice the solids.

LESSON INFORMATION

Student Learning Goals

- Let's analyze cross sections by slicing three-dimensional solids.

Student Learning Targets

- I can identify the three-dimensional shape that generates a set of cross sections.
- I can visualize and draw multiple cross sections of a three-dimensional figure.

Lesson Narrative

In grade 7, students described the two-dimensional figures that result from slicing three-dimensional figures. Here, these concepts are revisited with some added complexity. Students analyze **cross sections**, or the intersections between planes and solids, by slicing three-dimensional objects. Next, they identify three-dimensional solids given parallel cross-sectional slices. In addition, they revisit solid geometry vocabulary terms from earlier grades: **sphere**, **prism**, **cylinder**, **cone**, **pyramid**, and **faces**.

Spatial visualization in three dimensions is an important skill in mathematics. Understanding the relationship between solids and their parallel cross sections will be critical to understanding Cavalieri's Principle in later lessons. Cavalieri's Principle will be applied to the development of the formula for the volume of pyramids and cones. Students use spatial visualization to make sense of three-dimensional figures and their cross sections throughout the lesson (MP1).

WARM-UP| SLICE THIS *5 minutes*

Instructional Routines

- Think Pair Share

The purpose of this activity is for students to visualize what a cross section might look like and then test the prediction by observing the result of slicing through a solid. Cylindrical food items, such as cheese or carrots, are convenient examples.

LAUNCH

Arrange students in groups of 2. Tell students that a **cross section** is the intersection between a solid and a plane, or a two-dimensional figure that extends forever in all directions. Using a cylindrical food item such as cheese or carrots, or another cylindrical object, demonstrate that slicing a cylinder parallel to its base produces a circular cross section.

Then, give students quiet work time and then time to share their work with a partner.

STUDENT-FACING TASK STATEMENT

Imagine slicing a cylinder with a straight cut. The flat surface you sliced along is called a **cross section**. Try to sketch all the possible kinds of cross sections of a cylinder.

Possible Responses

The possible cross sections are a circle, a rectangle, an ellipse, and an ellipse with a part cut off.

Anticipated Misconceptions

Students may not consider non-horizontal or non-vertical cross sections at first. Remind them that a cross section is the intersection of any plane with a solid—the plane doesn't have to be vertical or horizontal.

Activity Synthesis

Ask students to share their predictions of what the cross sections will look like. Demonstrate slicing each cylindrical food item according to student instructions to see several examples.

EXPLORATION ACTIVITY | SLICE THAT *20 minutes*

Instructional Routines

- MLR7: Compare and Connect

In this activity, students continue to develop familiarity with three-dimensional solids and their cross sections. Students use spatial visualization to predict what cross sections might look like and then test their predictions.

This activity works best when each student has access to devices that can run the applet because students will benefit from seeing the relationship in a dynamic way. If students don't have individual access, projecting the applet would be helpful during the synthesis.

LAUNCH

Arrange students in groups of 3–4. Ask students to think about definitions of some geometric solids: spheres, prisms, pyramids, cones, and cylinders. Give students some quiet work time and then time to share their work with a partner. Follow with a whole-class discussion.

A **sphere** is the set of points in three-dimensional space the same distance from some center. A prism has two congruent **faces** (or sides) that are called bases. The bases are connected by quadrilaterals. A **cylinder** is like a prism except the bases are circles. A **pyramid** has one base. The remaining faces are triangles that all meet at a single vertex. A **cone** is like a pyramid except the base is a circle.

Give each group clay or playdough formed into the shape of a three-dimensional solid (cube, sphere, cylinder, cone, or other solids), and dental floss to slice the clay. Tell students that to view multiple cross sections, they will slice the shape, then re-form the shape and slice again.

An alternative is to find food items with interesting cross sections or three-dimensional foam solids from a craft store and providing plastic knives to slice the solids. In this case, provide each group with several of the same solid so they can experiment with multiple slices.

Try to include a sphere, a cube, and a cone in the collection of solids.

Support for English Language Learners

Representing, Conversing: MLR7 Compare and Connect. Use this routine to help students develop the mathematical language of cross sections of geometric solids. After students explore the cross sections of their solid, invite them to create a visual display of the cross sections they found. Then ask students to quietly circulate and observe at least two other visual displays in the room. Give students quiet think time to consider what is the same and what is different about their cross sections. Next, ask students to find a partner to discuss what they noticed. Listen for and amplify the language students use to compare and contrast various cross sections of a solid.

Design Principle(s): Cultivate conversation

Support for Students with Disabilities

Engagement: Develop Effort and Persistence. Connect a new concept to one with which students have experienced success. For example, remind students about the cross sections of a cylinder from the previous activity. Ask students how they created various cross sections of a cylinder such as a circle, rectangle, and an ellipse. Then ask students how they can apply this method to create various cross sections of their solid.

Supports accessibility for: Social-emotional skills; Conceptual processing

STUDENT-FACING TASK STATEMENT

Your teacher will give your group a three-dimensional solid to analyze.

1. Sketch predictions of all the kinds of cross sections that could be created from your solid.
2. Slice your solid to confirm your predictions. Sketch any new cross sections you find after slicing.

Possible Responses

Sample responses:

- cone cross sections: circles of different sizes, ellipses of different sizes, triangle, parabola
- sphere cross sections: circles of different sizes
- cube cross sections: a wide variety of shapes including triangles, quadrilaterals, pentagons, and hexagons

Anticipated misconceptions

If using the paper and pencil version of this activity and students are stuck, suggest they slice their solids at different angles and locations to see if different cross sections are generated.

Activity Synthesis

Invite groups of students with different solids to share their list of cross sections with the class. Ask students:

- “Were there any cross sections that caught you by surprise?” (It was surprising that a cube can have cross sections that are triangles, quadrilaterals, pentagons, and hexagons.)
- “Compare and contrast the different cross sections of a sphere.” (All the cross sections were circles, but they were different sizes.)
- “How are a cube’s cross sections different from a sphere’s?” (The cube has many differently-shaped cross sections, while the sphere’s cross sections are all circles.)

EXPLORATION ACTIVITY | STACK ’EM UP *10 minutes*

Instructional Routines

- MLR8: Discussion Supports

In the last activity, students started with solids and identified various cross sections. In this activity, students view three-dimensional slabs of a solid between parallel cross sections and try to determine what the original solid was. Being able to visualize the relationship between a solid and its cross sections is important to later work on Cavalieri’s Principle.

LAUNCH

Ask students, “What solid would a stack of all the same coins create?” Display a stack of quarters and note that it creates the shape of a cylinder. Then display, in order, a quarter, a nickel, a penny, and a dime. Ask, “What solid would a stack of coins decreasing in size create?” Make a stack with a few of each type of coin to make a solid that resembles a cone.

Support for English Language Learners

Speaking: MLR8 Discussion Supports. To help students respond to the questions for discussion, provide sentence frames such as: “The cross sections of ______ taken parallel to the base are ______ because....” As students share their responses, press for details by asking students how they know that the cross sections of prisms taken parallel to the base are congruent, while the cross sections of pyramids taken parallel to the base are similar. Ask students to distinguish the meanings of geometric congruence and similarity.

Design Principle(s): Support sense-making; Optimize output (for justification)

Support for Students with Disabilities

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation such as: "I noticed ________ so I….", "How do you know…?", "That could/couldn't be true because….", and "I agree/disagree because…."

Supports accessibility for: Language; Social-emotional skills

STUDENT-FACING TASK STATEMENT

Each question shows several parallel cross-sectional slabs of the same three-dimensional solid. Name each solid.

1.


2.


3.


Possible Responses

1. pyramid
2. cube
3. sphere

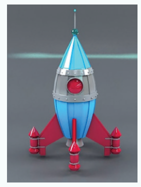
Activity Synthesis

Ask students to share their predictions for what solids are formed. Then display these images for all to see.

Now focus students' attention on cross sections that are taken parallel to a solid's base (for those solids that have bases). Ask students how cross sections can be used to differentiate between prisms and pyramids. (The cross sections of prisms taken parallel to the base are congruent to each other. The cross sections of pyramids taken parallel to the base are similar to each other.)

EXPLORATION EXTENSION | ARE YOU READY FOR MORE?

STUDENT-FACING TASK STATEMENT

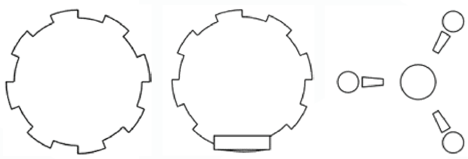


3D-printers stack layers of material to make a three-dimensional shape. Computer software slices a digital model of an object into layers, and the printer stacks those layers one on top of another to replicate the digital model in the real world.

1. Draw 3 different horizontal cross sections from the object in the image.
2. The layers can be printed in different thicknesses. How would the thickness of the layers affect the final appearance of the object?
3. Suppose we printed a rectangular prism. How would the thickness of the layers affect the final appearance of the prism?

Student Response

Sample responses:

1. 
2. Thicker layers would result in a chunkier look with more visible layers. Thinner layers would result in smoother curves.
3. All slices have edges perpendicular to the base of a printed object. Since a prism's edges are perpendicular to the base in reality, layer height does not affect the resulting print.

LESSON SYNTHESIS

In this lesson, students worked with three-dimensional solids and their **cross sections**. Here are questions for discussion:

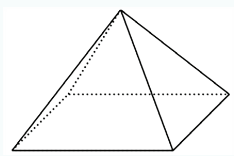
- "How are the cross sections in this lesson different from the two-dimensional figures we looked at in the last lesson?" (In the last lesson, we rotated the two-dimensional figures to trace out a solid. The two-dimensional figures were usually an outline of half of the figure, and they had to have a relationship to the axis of rotation of the solid. Here, our cross sections cut through the entire solid, and they can come from anywhere in the solid.)
- "What kinds of applications of cross sections might we see in real life?" (There is a field of medicine called tomography that is about finding ways to get images of cross sections of people. Technologies like the CAT scan, the MRI, and the PET scan allow doctors to examine cross sections of a brain, a lung, or an injury and visualize what the three-dimensional body part looks like.)

COOL DOWN | SKETCH IT *5 minutes*

LAUNCH

Consider providing a physical square pyramid for students who would find that helpful.

STUDENT-FACING TASK STATEMENT



Here is a square pyramid.

For each plane described, sketch the cross section that results from the intersection of the pyramid and the plane.

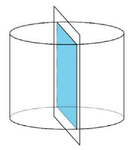
1. a horizontal plane
2. a vertical plane passing through the point at the top of the pyramid
3. a diagonal plane

Possible Responses

1. Students should sketch a square.
2. Students should sketch an isosceles triangle.
3. Students should sketch an isosceles trapezoid or another angled cross section of the pyramid.

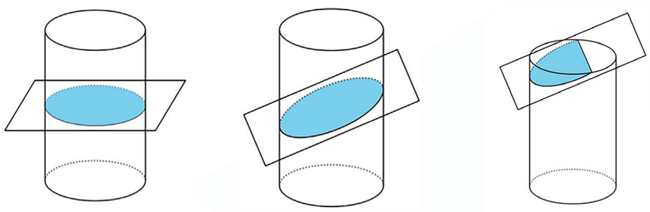
STUDENT LESSON SUMMARY

In earlier grades, you learned some vocabulary terms about solid geometry: A **sphere** is the set of points in three-dimensional space the same distance from some center. A **prism** has two congruent **faces** (or sides) that are called bases. The bases are connected by parallelograms. A **cylinder** is like a prism except the bases are circles. A **pyramid** has one base. The remaining faces are triangles that all meet at a single vertex. A **cone** is like a pyramid except the base is a circle.



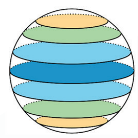
We often analyze **cross sections** of solids. A cross section is the intersection of a solid with a *plane*, or a two-dimensional figure that extends forever in all directions. For example, some cheese is sold in cylindrical blocks. If you stand the cheese on end and slice vertically, you will get a rectangle, as shown. This rectangle is a cross section of the cylinder.

Here are 3 more examples of cross sections created by intersecting a plane and a cylinder.



If you wanted to serve your cylindrical cheese at a party, you might cut it into several pieces, like this. The pieces are thin cylinders. They are like cross sections, but they are three-dimensional. All the cuts were made parallel to one another. By looking at the slices, or by stacking them up, you could figure out that the original shape of the cheese was a cylinder.

What if another cheese plate contained slices whose radii got bigger to a maximum size and then got smaller again? The cheese was probably in the shape of a sphere. A sphere has circular cross sections. The size of the circular cross sections increases as you get closer to the center of the sphere, then decreases past the center.



PRACTICE PROBLEMS

PROBLEM 1

Select **all** figures for which there exists a direction such that all cross sections taken at that direction are congruent.

- **A.** triangular pyramid
- **B.** square pyramid
- **C.** rectangular prism
- **D.** cube
- **E.** cone
- **F.** cylinder
- **G.** sphere

Possible Solutions

C, D, F

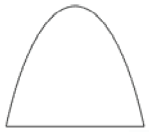
PROBLEM 2

Imagine an upright cone with its base resting on your horizontal desk. Sketch the cross section formed by intersecting each plane with the cone.

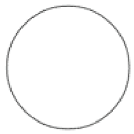
- **a.** vertical plane not passing through the cone's topmost point
- **b.** horizontal plane
- **c.** diagonal plane

Possible Solutions

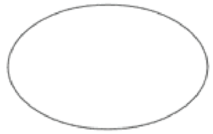
a.



b.



c.



PROBLEM 3

Name 2 figures for which a circle can be a cross section.

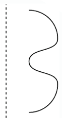
Possible Solutions

Sample responses: sphere, cone

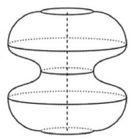
PROBLEM 4

(From Unit 5, Lesson 1.)

Sketch the solid of rotation formed by rotating the given two-dimensional figure using the dashed vertical line as an axis of rotation.



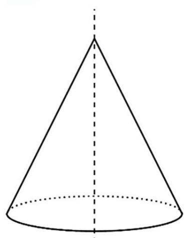
Possible Solutions



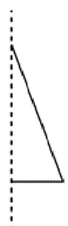
PROBLEM 5

(From Unit 5, Lesson 1.)

Draw a two-dimensional figure that could be rotated using a vertical axis of rotation to give the cone shown.



Possible Solutions



PROBLEM 6

(From Unit 4, Lesson 11.)

A regular hexagon and a regular octagon are both inscribed in the same circle. Which of these statements is true?

- **A.** The perimeter of the hexagon is less than the perimeter of the octagon, and each perimeter is less than the circumference of the circle.
- **B.** The perimeter of the octagon is less than the perimeter of the hexagon, and each perimeter is less than the circumference of the circle.
- **C.** The perimeter of the hexagon is greater than the perimeter of the octagon, and each perimeter is greater than the circumference of the circle.
- **D.** The perimeter of the octagon is greater than the perimeter of the hexagon, and each perimeter is greater than the circumference of the circle.

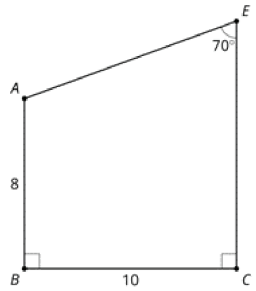
Possible Solutions

A

PROBLEM 7

(From Unit 4, Lesson 10.)

Technology required. Find the perimeter of the figure.



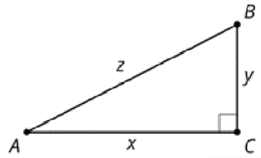
Possible Solutions

$26 + \frac{10}{tan(70)} + \frac{10}{sin(70)} \approx 40.3$ *units*

PROBLEM 8

(From Unit 4, Lesson 6.)

Match each trigonometric function to a ratio. You may use ratios more than once.



- **A.** $\tan(A)$
- **B.** $\tan(B)$
- **C.** $\cos(A)$
- **D.** $\cos(B)$
- **E.** $\sin(A)$
- **F.** $\sin(B)$

1. $\frac{y}{z}$
2. $\frac{x}{z}$
3. $\frac{x}{y}$
4. $\frac{y}{x}$

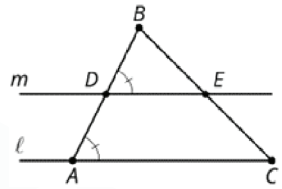
Possible Solutions

- A: 4
- B: 3
- C: 2
- D: 1
- E: 1
- F: 2

PROBLEM 9

(From Unit 1, Lesson 20.)

Explain how you know lines m and l are parallel.



Possible Solutions

Sample response: Angles BDE and BAC are congruent and corresponding. So, line m is parallel to line ℓ because these lines are cut by transversal AB and the corresponding angles are congruent.

REFLECTION AND NOTES

Unit 5, Lesson 2: Slicing Solids

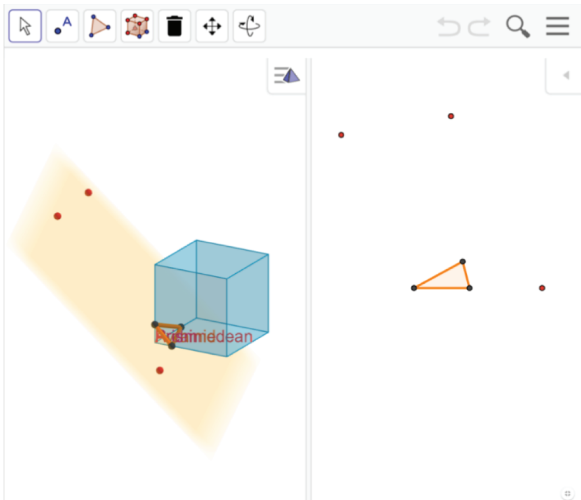
Slice This

Imagine slicing a cylinder with a straight cut. The flat surface you sliced along is called a **cross section**. Try to sketch all the possible kinds of cross sections of a cylinder.

Slice That

This activity requires the use of an applet, so please make your way over to the digital platform to find the link.

The triangle is a cross section formed when the plane slices through the cube.



1. Sketch predictions of all the kinds of cross sections that could be created as the plane moves through the cube.

2. The 3 red points control the movement of the plane. Click on them to move them up and down or side to side. You will see one of these movement arrows appear. Sketch any new cross sections you find after slicing.

Stack 'Em Up

Each question shows several parallel cross-sectional slabs of the same three-dimensional solid. Name each solid.

1. 

2. 

3. 

Practice Problems

1. Select **all** figures for which there exists a direction such that all cross sections taken at that direction are congruent.

   A. triangular pyramid

   B. square pyramid

   C. rectangular prism

   D. cube

   E. cone

   F. cylinder

   G. sphere

2. Imagine an upright cone with its base resting on your horizontal desk. Sketch the cross section formed by intersecting each plane with the cone.

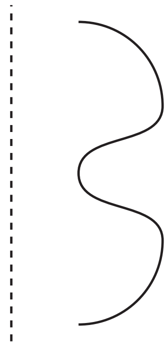
   A. vertical plane not passing through the cone's topmost point

   B. horizontal plane

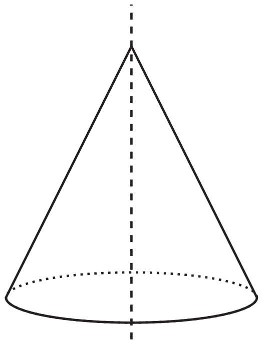
   C. diagonal plane

3. Name 2 figures for which a circle can be a cross section.

4. Sketch the solid of rotation formed by rotating the given two-dimensional figure using the dashed vertical line as an axis of rotation.



5. Draw a two-dimensional figure that could be rotated using a vertical axis of rotation to give the cone shown.



6. A regular hexagon and a regular octagon are both inscribed in the same circle. Which of these statements is true?

   A. The perimeter of the hexagon is less than the perimeter of the octagon, and each perimeter is less than the circumference of the circle.

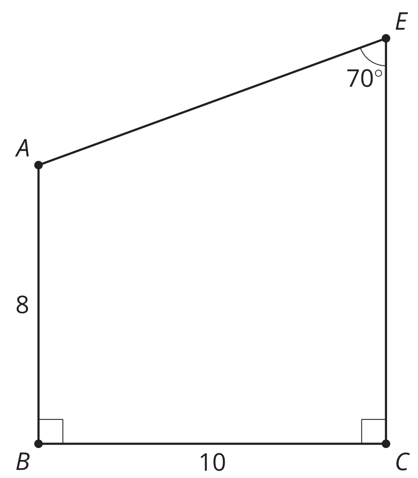
   B. The perimeter of the octagon is less than the perimeter of the hexagon, and each perimeter is less than the circumference of the circle.

   C. The perimeter of the hexagon is greater than the perimeter of the octagon, and each perimeter is greater than the circumference of the circle.

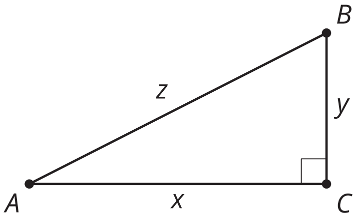
   D. The perimeter of the octagon is greater than the perimeter of the hexagon, and each perimeter is greater than the circumference of the circle.

7. *Technology required*. Find the perimeter of the figure.

This activity requires the use of an applet, so please make your way over to the digital platform to find the link.



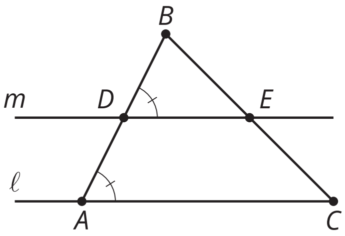
8. Match each trigonometric function to a ratio. You may use ratios more than once.



A. $\tan(A)$
B. $\tan(B)$
C. $\cos(A)$
D. $\cos(B)$
E. $\sin(A)$
F. $\sin(B)$

1. $\frac{y}{z}$
2. $\frac{x}{z}$
3. $\frac{x}{y}$
4. $\frac{y}{x}$

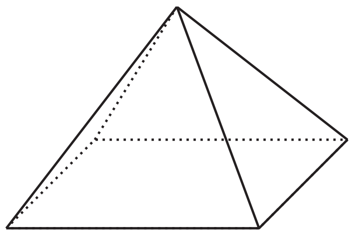
9. Explain how you know lines m and l are parallel.



Lesson 2: Slicing Solids

Cool Down: Sketch It

Here is a square pyramid.



For each plane described, sketch the cross section that results from the intersection of the pyramid and the plane.

1. a horizontal plane

2. a vertical plane passing through the point at the top of the pyramid

3. a diagonal plane

Notes

Ready to see the full program?

Scan here!

Exploring this lesson with your students?

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By sharing, you'll join a community of math educators who are making math meaningful. You might even get featured!