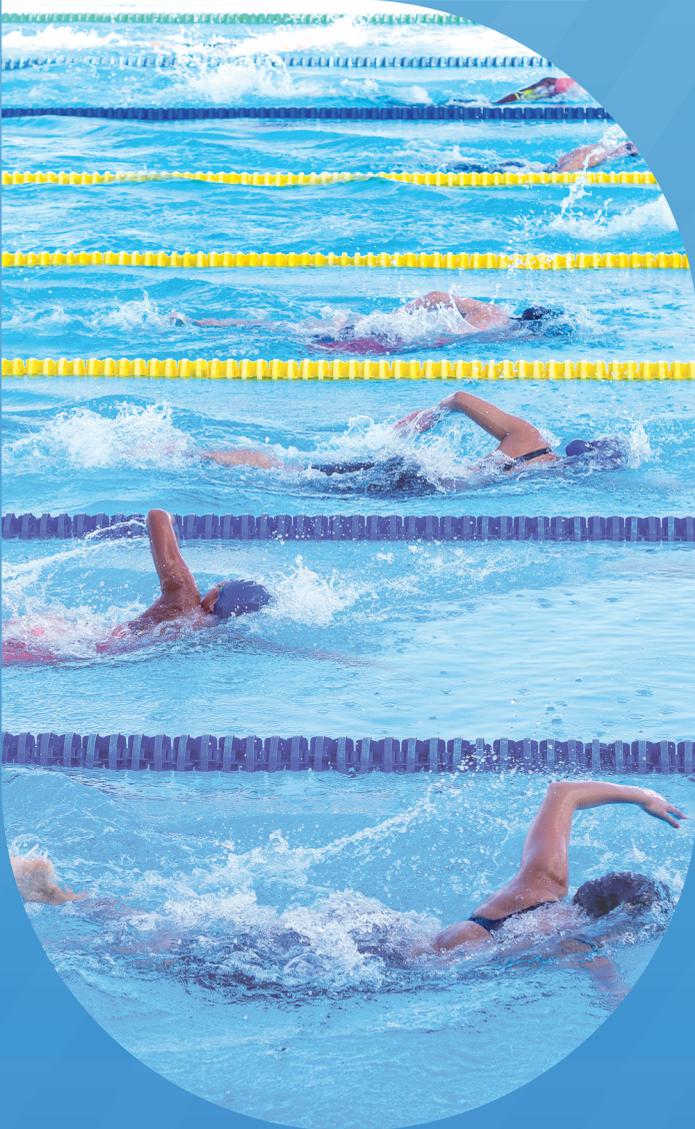


Algebra 1

MATH NATION

BY ACCELERATE LEARNING

Take & Teach



*Accelerate*LEARNING
THE LEADER in STEM EDUCATION

What's Inside This Sample Lesson?

- A fully guided lesson written to meet rigorous state and national standards
- **Teacher Edition** pages, **Student Workbook** pages, and other **helpful resources** to fully experience a Math Nation lesson
- Warm-ups, exploration tasks, instructional routines, and teacher prompts
- Support for English learners and students with disabilities
- Integrated reflection, synthesis, and cool-down opportunities

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Go Online!

Explore the digital resources for this lesson.



LESSON 5

FITTING LINES

LEARNING GOALS

- Determine which linear model is a better fit for a given data set.
- Interpret (orally and in writing) the rate of change and vertical intercept for a linear model in everyday language.
- Use technology to generate the line of best fit, and use the equation representing the linear model to predict (extrapolate) and estimate (interpolate) values not given in the data set.

ALIGNMENT

Building On

- **8.SP.A.2** Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

Addressing

- **HSS-ID.B.6.c** Fit a linear function for a scatter plot that suggests a linear association.
- **HSS-ID.C.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

Building Toward

- **HSS-ID.B.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
- **HSS-ID.B.6.b** Informally assess the fit of a function by plotting and analyzing residuals.

LESSON PREPARATION

Required Materials

- Graphing technology
- Pre-printed slips, cut from copies of the blackline master

Required Preparation

- Print and cut up slips for the card sort. One copy of the blackline master for every group of 2 students. To find best fit lines, students will need access to technology that will compute the least-squares regression lines for a set of data.
- Acquire devices that can run Desmos (recommended) or other graphing technology. It is ideal if each student has their own device. (Desmos is available under Math Tools.)

LESSON INFORMATION

Student Learning Goal

- Let's find the best linear model for some data.

Student Learning Targets

- I can describe the rate of change and y -intercept for a linear model in everyday language.
- I can use technology to find the line of best fit.

Lesson Narrative

The mathematical purpose of this activity is for student to informally assess the fit of various lines to data, to use technology to find the line of best fit, and to interpret the slope and vertical intercept of the linear model. The work connects to previous work because students created scatter plots and created linear models for the data. The work connects to upcoming work because students will use the correlation coefficient to describe the strength of the linear relationship. When students sort scatter plots into linear and nonlinear categories, and when they organize lines of best fit by the goodness of fit, they look for and make use of structure (MP7), because they are analyzing representations and structures closely, and making connections. Students are reasoning abstractly and quantitatively (MP2) when they interpret the meaning of the slope and vertical intercept in context.

WARM-UP | SELECTING THE BEST LINE 5 minutes

The mathematical purpose of this activity is for students to be able to visually assess the best line that fits data among a set of choices. Students are given a scatter plot and 2 lines that may fit the data. Students must select the line that better fits the data. The given lines address many common errors in student thinking about best fit lines including: going through the most points, dividing the data in half, and connecting the points on both ends of the scatter plot.

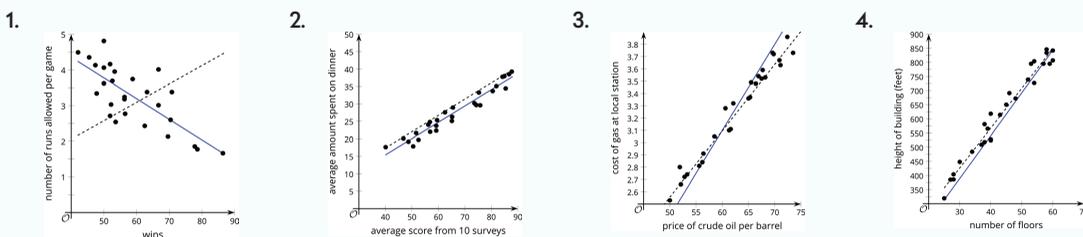
Listen for students using the terms slope and y -intercept.

LAUNCH

Provide students access to the images. Give students 2 minutes of quiet time to work the questions.

STUDENT-FACING TASK STATEMENT

Which of the lines is the best fit for the data in each scatter plot? Explain your reasoning.



Possible Responses

1. Sample response: The solid line is the better fit for the data since it goes through the middle of the data and follows the negative trend of the data.
2. Sample response: The solid line is the better fit for the data since it goes through the middle of the data with approximately equal numbers of data points on either side of the line.
3. Sample response: The dashed line is the better fit for the data since it goes through the middle of the data with a slope that follows the trend of the data. The solid line has a lot of points above the line to start and a lot of points below the line at the end.

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4. Sample response: The dashed line is the better fit for the data since it goes through the middle of the data, follows the trend of the data, and has a similar number of points on each side. The solid line fits the first and last point perfectly, but most of the data is above the line.

Activity Synthesis

The purpose of this discussion is to understand bad fit, good fit, and best fit. In each scatter plot, the solid line represents the line of best fit—except for the last two graphs, for which the dashed line is the best fit.

Ask a student who uses the term slope while working the questions, “Can you explain the relationship between the two lines in question about runs and wins using the concept of slope?” (The slope of the dashed line is positive and the slope of the solid line is negative.)

Ask a student who uses the term y -intercept, “Can you explain the significance of the y -intercept in the question about average survey scores and amount spent on dinner?” (The solid line will have a y -intercept less than the y -intercept for the dashed line. Because the two lines have approximately the same slope, they appear parallel in the scatter plot.)

If time permits, discuss questions such as:

- “Is the dashed line in question about runs and wins a bad fit, good fit, or best fit?” (The line is a bad fit because it does not show the correct relationship between the variables. It shows that the value of y increases as the value of x increases, rather than the value of y decreasing as the value of x increases.)
- “Is the dashed line in question about oil and gas a bad fit, good fit, or best fit?” (It is the best fit because it is close to going through the middle of the data and follows the same trend as the data.)
- “What factors helped you select the linear model that fits the data best?” (The line should go through the middle of the data, follow the trend of the data, and have a similar number of points on each side of the line.)

EXPLORATION ACTIVITY | CARD SORT: DATA PATTERNS *15 minutes*

Instructional Routines

- Card Sort
- MLR8: Discussion Supports
- Take Turns

The mathematical purpose of this activity is for students to:

- distinguish between linear and nonlinear relationships in bivariate, numerical data
- informally assess the fit of a linear model
- compare the slope and the vertical intercepts of different linear models
- describe the relationship between two variables.

Students are given cards showing scatter plots and a linear model. They sort the cards based on how well the lines fit the data as well as by slope and intercept in increasing order.

A sorting task gives students opportunities to analyze representations, statements, and structures closely and make connections (MP2, MP7).

LAUNCH

Arrange students in groups of 2. Give students a chance to familiarize themselves with what is on the cards. For example, you might ask them to sort the cards into categories of their choosing, and explain their categories to their partner.

Support for English Language Learners

Conversing: MLR8 Discussion Supports. In pairs, ask students to take turns sorting the cards and explaining their reasoning to their partner. Display the following sentence frames for all to see: “_____ should be before _____ because ...”, and “I noticed _____, so I ...”. Encourage students to challenge each other when they disagree. This will help students clarify their reasoning about linear models.

Design Principle(s): Support sense-making; Maximize meta-awareness

Support for Students with Disabilities

Action and Expression: Internalize Executive Functions. Provide students with a template for organizing their observations. Provide a template or invite students to fold a blank piece of paper in thirds, and label with three headers of “ x ,” “ y ” and “linear model fits?” to collect their answers. Explain that in the first column, they will always write increasing, since they will be reading each graph from left to right, then have them fill in the behavior of the y values in the next column, and in the last column, their conclusions about whether a linear model fits well.

Supports accessibility for: Language; Organization

STUDENT-FACING TASK STATEMENT

Your teacher will give you a set of cards that show scatter plots.

- Arrange all the cards in three different ways. Ensure that you and your partner agree on the arrangement before moving on to the next one. Sort all the cards in order from.
 - best to worst for representing with a linear model
 - least to greatest slope of a linear model that fits the data well
 - least to greatest vertical intercept of a linear model that fits the data well
- For each card, write a sentence that describes how y changes as x increases and whether the linear model is a good fit for the data or not.

Possible Responses

- An order for each sort is:
 - Sample response: B, E, A and F, C, D and G. There may be some debate about the exact order. It is important that students recognize that B is the best fit, D and G should not be fit with linear models, and be able to reason that linear models fit more poorly the farther the data are from the line.
 - E, B, A and F, C. D and G do not have linear models that fit the data well.
 - A, C, F, B, E. D and G do not have linear models that fit the data well.
- Sample responses:
 - As x increases, y also increases. The linear model fits the data fairly well.
 - As x increases, y decreases. The linear model fits the data very well.
 - As x increases, y increases. The linear model fits most of the data very well; however, there are two values that do not follow this trend.
 - As x increases, y goes up and down like a wave. The data does not follow the line very well at all.
 - As x increases, y decreases. Most of the data fits a linear model very well, but the one point at (7,15) does not fit with the rest of the data very well.
 - As x increases, y increases. The linear model fits the data fairly well.
 - As x increases, y decreases and then increases. The data does not follow the line very well at all

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Activity Synthesis

The purpose of this discussion is for students to discuss the goodness of fit for linear models.

Here are some questions for discussion.

- “How are scatter plots of A and F the same? How are they different?” (They have the same slope and the linear model for each scatter plot are equally well fit. They are different because they have a different vertical intercept.)
- “How do you know if a linear model is a good fit?” (You need to look at the scatter plot and the line of best fit and make a decision about whether or not the data follows a linear trend.)
- “Why is the goodness fit for the linear model in scatter plot B better than the fit for the linear model in scatter plot A?” (The data in B falls on or very close to the linear model. The data in A is scattered around the line of best fit and has roughly the same number of values below that line of best fit as it does above the line of best fit.)

EXPLORATION ACTIVITY | FITTING LINES WITH TECHNOLOGY *15 minutes*

Instructional Routines

- Fit It
- MLR7: Compare and Connect

It is recommended to use the digital version of this activity. The mathematical purpose of this activity is for students to use technology to compute a line of best fit for data given in a table, and to understand the meaning of the slope and y -intercept. If the digital version of the activity is not available, the students should be guided through using available technology to find the least-squares regression line as the line of best fit.

LAUNCH

Provide data tables for the graphs for the cards from the previous activity that were fit well with a linear model. Assign one table to each group. For students using the paper task, show them how to use technology to create a scatter plot of the data in a table. After groups have had a chance to estimate the best fit lines, pause the class. Show students how to use technology to find the least-squares regression line for data and display the line with the scatter plot.

Display the tables for students to use for the last question:

A. (card A in the previous activity)		B. (card B in the previous activity)		C. (card C in the previous activity)		D. (card E in the previous activity)		E. (card F in the previous activity)	
x	y	x	y	x	y	x	y	x	y
1	2	1	11.86	1	6.11	1	13.9	1	6.5
2.2	4	2.2	11.332	2.2	7.142	2.2	11.5	2.2	8.5
3.3	5	3.3	10.848	3.3	8.088	3.3	9.3	3.3	9.5
3.3	4.5	3.3	10.741	3.3	8.19	3.3	9.2	3.3	9
3.6	6	3.6	10.716	3.6	8.346	3.6	8.7	3.6	10.5
3.8	6.4	3.8	10.628	3.8	2.92	3.8	8.3	3.8	11
3.9	5.7	3.9	10.584	3.9	8.604	3.9	8.1	3.9	10.2
4	7	4	10.54	4	8.69	4	7.9	4	11.5
4.4	6.5	4.4	10.364	4.4	9.034	4.4	7.1	4.4	11

4.5	7	4.5	10.32	4.5	9.12	4.5	6.9	4.5	11.5
4.7	7	4.7	10.232	4.7	9.292	4.7	6.5	4.7	11.5
4.8	6	4.8	10.188	4.8	13.6	4.8	6.3	4.8	10.5
4.9	8.7	4.9	10.144	4.9	9.464	4.9	6.1	4.9	13.2
5	7	5	10.1	5	9.55	5	5.9	5	11.5
5.1	7.7	5.1	10.056	5.1	9.636	5.1	5.7	5.1	12.2
5.2	6.7	5.2	10.5	5.2	9.722	5.2	5.5	5.2	11.2
5.5	8	5.5	9.88	5.5	9.98	5.5	4.9	5.5	12.5
5.5	8.5	5.5	9.753	5.5	10.32	5.5	4.3	5.5	13
6	9.5	6	9.66	6	10.41	6	3.9	6	14
6.6	8.6	6.6	9.396	6.6	10.926	6.6	1.3	6.6	13.1
7	9	7	9.22	7	11.27	7	1.9	7	13.5
7.7	10.313	7.7	8.912	7.7	11.872	7.7	0.5	7.7	14.813

Support for Students with Disabilities

Engagement: Develop Effort and Persistence. Provide prompts, reminders, or checklists that focus on increasing the length of on-task orientation in the face of distractions. For example, provide two copies of the steps: graph the table, find the best fit line, find the slope, and find the y -intercept. Include phrases to activate knowledge from prior activities such as, "As x increases ..."

Supports accessibility for: Organization; Conceptual processing; Attention

STUDENT-FACING TASK STATEMENT

The weight of ice cream sold in a day at a small store in pounds (x) and the average temperature outside during the day in degrees Celsius (y) are recorded in the table.



x	20	18	21	17	21.5	19.5	21	18
y	6	4.5	6.5	3.5	7.5	6.5	7	5

- For this data, create a scatter plot and sketch a line that fits the data well.
- Use technology to compute the best fit line. Round any numbers to 2 decimal places.
- What are the values for the slope and y -intercept for the best fit line? What do these values mean in this situation?
- Use the best fit line to predict the y value when x is 10. Is this a good estimate for the data? Explain your reasoning.
- Your teacher will give you a data table for one of the other scatter plots from the previous activity. Use technology and this table of data to create a scatter plot that also shows the line of best fit, then interpret the slope and y -intercept.

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Possible Responses

- Answers vary. Lines should go through the middle of the data.
- $y = 0.78x - 9.44$
- The slope is 0.78, which means that about one additional pound of ice cream is sold with every 0.78-degree increase in the outside temperature. The y -intercept is $(0, -9.44)$, which means that no ice cream is sold when the temperature is about -9.44 degrees Celsius.
- -1.64 degrees Celsius. Sample explanation: This may be a good estimate for the data, since the points seem close to the line, but something different might happen when the temperature gets below freezing, so it's possible that when only 10 pounds of ice cream are sold, the relationship with the weather may be different.
- $y = 1.1979x + 1.3196$. When x is zero, y is 1.3196. As x increases by 1, y increases, on average, by 1.1979.
 - $y = -0.4337x + 12.288$. When x is zero, y is 12.288. As x increases by 1, y decreases, on average, by 0.4337.
 - $y = 0.9749x + 4.6529$. When x is zero, y is 4.6529. As x increases by 1, y increases, on average, by 0.9749.
 - $y = -2.063x + 16.144$. When x is zero, y is 16.144. As x increases by 1, y decreases, on average, by 2.063.
 - $y = 1.1979x + 5.8196$. When x is zero, y is 5.8196. As x increases by 1, y increases, on average, by 1.1979.

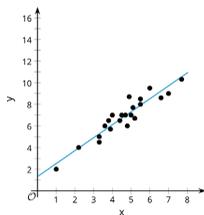
Anticipated misconceptions

Students may struggle with interpreting slope and y -intercept. Remind students of how each relates to a situation. To help students interpret slope, ask them: "What does the x variable represent? What does the y variable represent? How is slope connected to the x and y variables? What happens to x as y increases (or decreases)?" To help students interpret the y -intercept, ask them: "What does the point $(20,6)$ mean in the scatter plot? What are the coordinates of the y -intercept? What do each of the coordinates mean in the situation described? What is the y value when x is 0? Which variable has a value of 0? Which variable is represented with y ?"

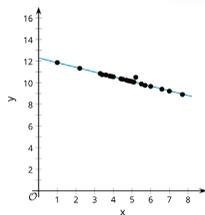
Activity Synthesis

The purpose of this discussion is for students to make connections between the scatter plot and the equation of the line of best fit. Display each scatter plot, the line of best fit, and the equation of the line of best fit.

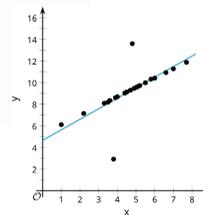
A. $y = 1.979x + 1.3196$



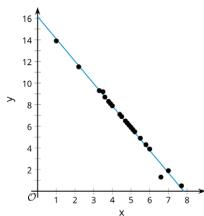
B. $y = -0.4337x + 12.288$



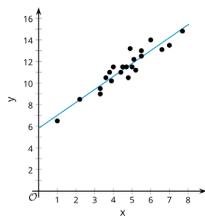
C. $y = 0.9749x + 4.6529$



D. $y = -2.063x + 16.144$



E. $y = 1.1979x + 5.8196$



Here are some questions for discussion:

- “How does using technology help model the data in the scatter plot?” (It allows different people to come up with the same equation for the line of best fit. If the line is just drawn by hand, there can be different linear equations that seem to fit the data well, but there is only one “best” fit line.)
- “What does the y -intercept represent in each scatter plot? When is it reasonable to use this interpretation?” (It represents the value of y estimated by the linear model when $x = 0$. When the intercept is near the range of the data, it can be reasonable to use this interpretation because otherwise, the linear trend may not continue. There are also some situations in which a value of 0 for x does not make sense.)
- “Why is the slope the same in scatter plot A and scatter plot F?” (It is the same because the data in scatter plot F is the same data as in scatter plot A, except that the values for y have all been increased by 4.5 units.)

Tell students they should be careful when predicting values outside the range of the data, in particular, for the y -intercept. Even when the data is fit well by a linear model, the behavior of the variables farther away may not be linear. It is important to remember that all predictions using the best fit line are estimates and the reasonableness of the predictions should be considered.

Support for English Language Learners

Representing, Conversing: MLR7 Compare and Connect. As students share the connections they noticed between the scatter plot and the equation of the line of best fit, call students’ attention to the different ways the slope and vertical intercept are represented. Wherever possible, amplify student words and actions that describe the connections between a specific feature of one mathematical representation and a specific feature of another representation.

Design Principle(s): Maximize meta-awareness; Support sense-making

EXPLORATION EXTENSION | ARE YOU READY FOR MORE?

STUDENT-FACING TASK STATEMENT

Priya uses several different ride services to get around her city. The table shows the distance, in miles, she traveled during her last 10 trips and the price of each trip, in dollars.

1. Priya creates a scatter plot of the data using the distance, x , and the price, y . She determines that a linear model is appropriate to use with the data. Use technology to find the equation of a line of best fit.
2. Interpret the slope and the y -intercept of the equation of the line of best fit in this situation.
3. Use the line of best fit to estimate the cost of a 3.6-mile trip. Will this estimate be close to the actual value? Explain your reasoning.
4. On her next trip, Priya tries a new ride service and travels 3.6 miles, but pays only \$4.00 because she receives a discount. Include this trip in the table and calculate the equation of the line of best fit for the 11 trips. Did the slope of the equation of the line of best fit increase, decrease, or stay the same? Why? Explain your reasoning.
5. Priya uses the new ride service for her 12th trip. She travels 4.1 miles and is charged \$24.75. How do you think the slope of the equation of the line of best fit will change when this 12th trip is added to the table?

Distance (miles)	Price (\$)
3.1	12.5
4.2	14.75
5	16
3.5	13.25
2.5	12
1	9
0.8	8.75
1.6	9.75
4.3	12
3.3	14

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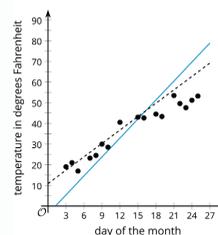
Possible Responses

- $y = 1.57x + 7.6$
- Sample response: The slope means that the fare increases by \$1.57 for each additional mile driven. The y-intercept means that the fee for getting in the car is approximately \$7.60.
- Sample response: According to the line of best fit, the cost of a 3.6-mile trip is approximately \$13.25, which is the same price as the 3.5-mile trip listed in the table.
- Sample response: The slope of the line decreased to \$1.27 for each additional mile. It probably decreased because \$4.00 is a far lower price per mile than any of her other trips because of the discount. This causes the slope, the increase in price for each additional mile, to decrease.
- Sample response: The slope of the line will probably increase, because \$24.75 is very expensive and will likely cause the cost for each additional mile to increase.

LESSON SYNTHESIS

Here are some questions for discussion.

- “What is a line of best fit?” (The best linear model for the data.)
- “How do you know that you have a line of best fit?” (You can use technology to generate the line of best fit, but then you need to graph it on the scatter plot to verify that it fits the data well. It needs to follow the trend of the data, it should go roughly through the middle of the data, and it should have roughly the same number of data points on either side of it.)
- “Which line fits the data better, the solid line or the dashed line? Do you think it is the line of best fit? Explain your reasoning.” (The dashed line fits the data better, because its slope and vertical intercept more closely resemble the trend of the data than the slope and vertical intercept of the solid line. It is not the line of best fit because it does not go through the middle of the data. It should be a little lower on the graph.)

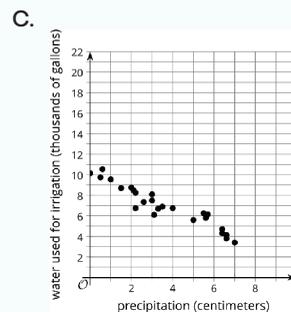
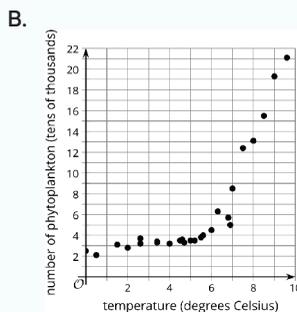
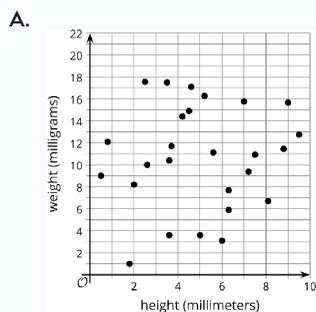


COOL DOWN | FRESH AIR 5 minutes

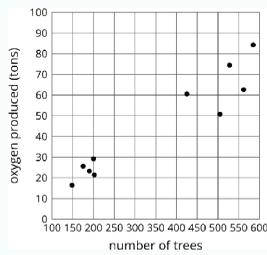
If possible, provide access to devices that can run Desmos or other graphing technology.

STUDENT-FACING TASK STATEMENT

- Which of these scatter plots shows data that would best be modeled with a linear function? Explain your reasoning.



2. Which of the lines is most likely the line of best fit for the data provided?



- A. $y = -0.51x + 225.12$
- B. $y = 0.34x - 34.05$
- C. $y = 0.13x - 0.19$
- D. $y = 0.98x - 21.13$

Number of Trees in a Forest	Tons of Oxygen Produced by the Forest
148	16.43
175	25.64
190	23.28
200	29.2
202	21.41
425	60.56
505	50.75
528	74.45
562	62.66
585	84.24

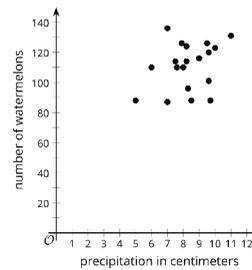
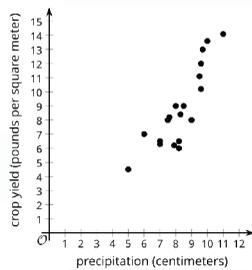
Possible Responses

- Sample reasoning: The points in the scatter plot C seem to follow the shape of a line best. The scatter plot in A is very spread out and the scatter plot in B is probably better fit by a curve.
- $y = 0.13x - 0.19$

STUDENT LESSON SUMMARY

Some data appear to have a linear relationship, so finding an equation for a line that fits the data can help you understand the relationship between the variables.

Other data may follow non-linear trends or not have an apparent trend at all.



When modeling data with a linear function seems useful, it is important to find a linear function that is close to the data. The line should have a y -intercept and slope to follow the shape of the data in the scatter plot as much as possible.

Technology can be used to quickly find a line of best fit for the data and provide the equation of the line that we can use to analyze the situation.

Algebra 1 Mathematics

PRACTICE PROBLEMS

PROBLEM 1

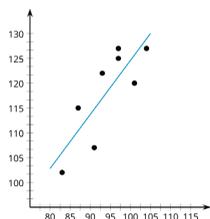
Technology required.

- A. Use graphing technology to create a scatter plot and find the best fit line.
- B. What does the best fit line estimate for the y value when x is 100?

x	83	87	91	93	97	97	101	104
y	102	115	107	122	125	127	120	127

Possible Solutions

- 1. $y = 1.09x + 15.59$
- 2. 124.59



PROBLEM 2

Technology required.

- A. What is the equation of the line of best fit? Round numbers to 2 decimal places.
- B. What does the equation estimate for y when x is 2.3? Round to 3 decimal places.
- C. How does the estimated value compare to the actual value from the table when x is 2.3?
- D. How does the estimated value compare to the actual value from the table when x is 3?

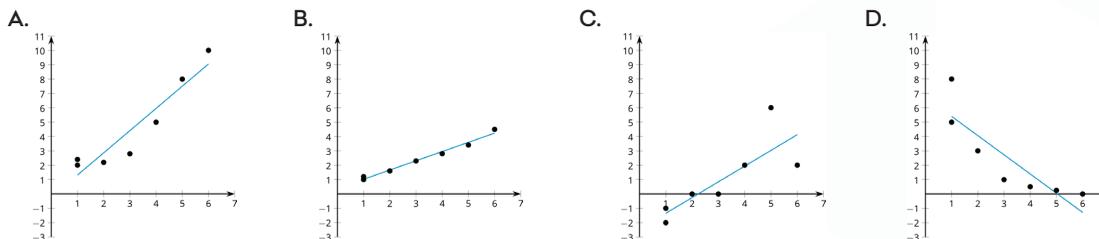
x	2.3	2.8	3.1	3	3.5	3.8
y	6.2	5.7	4.7	3.2	3	2.8

Possible Solutions

- A. $y = -2.45x + 11.83$
- B. 6.195
- C. Sample response: The values are very close, differing only by 0.005.
- D. Sample response: The estimated value is 4.48 since $-2.45 \cdot 3 + 11.83 = 4.48$, and the actual value is 3.2. These are different by 1.28, which is relatively large compared to the other values.

PROBLEM 3

Which of these scatter plots are best fit by the shown linear model?



Possible Solutions

B

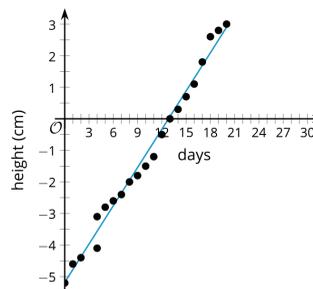
PROBLEM 4

(From Unit 3, Lesson 4)

A seed is planted in a glass pot and its height is measured in centimeters every day.

The best fit line is given by the equation $y = 0.404x - 5.18$, where y represents the height of the plant above ground level, and x represents the number of days since it was planted.

- A. What is the slope of the best fit line? What does the slope of the line mean in this situation? Is it reasonable?
- B. What is the y -intercept of the best fit line? What does the y -intercept of the line mean in this situation? Is it reasonable?



Possible Solutions

Sample responses:

- A. **0.404.** It means that each day the plant grows about 0.404 centimeters taller. This is fairly reasonable since a young plant should grow, and this number is reasonable for the amount a young plant grows.
- B. **(0, -5.18).** It means that on day zero, when the seed was planted, it was 5.18 centimeters below the surface of the ground. This is reasonable since seeds are planted below the surface, and about 5 centimeters seems like the right amount.

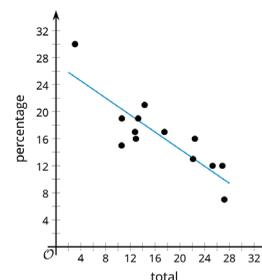
PROBLEM 5

(From Unit 3, Lesson 4)

At a restaurant, the total bill and the percentage of the bill left as a tip are represented in the scatter plot.

The best fit line is represented by the equation $y = -0.632x + 27.1$, where x represents the total bill in dollars, and y represents the percentage of the bill left as a tip.

- A. What does the best fit line estimate for the percentage of the bill left as a tip when the bill is \$15? Is this reasonable?
- B. What does the best fit line predict for the percentage of the bill left as a tip when the bill is \$50? Is this reasonable?



Possible Solutions

- A. **17.62%** since $-0.632 \cdot 15 + 27.1 = 17.62$. This is reasonable based on the data, since it is consistent with the rest of the data. It is also customary to tip around 15% to 20% of the bill and this value falls within that range.
- B. **-4.5%** since $-0.632 \cdot 50 + 27.1 = -4.5$. This is not reasonable, since a negative value does not make sense for a percentage of the bill to leave as a tip.

PROBLEM 6

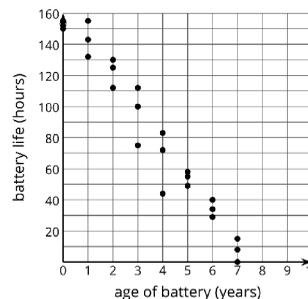
(From Unit 3, Lesson 4)

A recent study investigated the amount of battery life remaining in alkaline batteries of different ages. The scatter-plot shows this relationship between the different alkaline batteries tested.

The scatter plot includes a point at (7,15). Describe the meaning of this point in this situation.

Possible Solutions

Sample Response: A 7-year-old battery has 15 hours of battery life remaining.

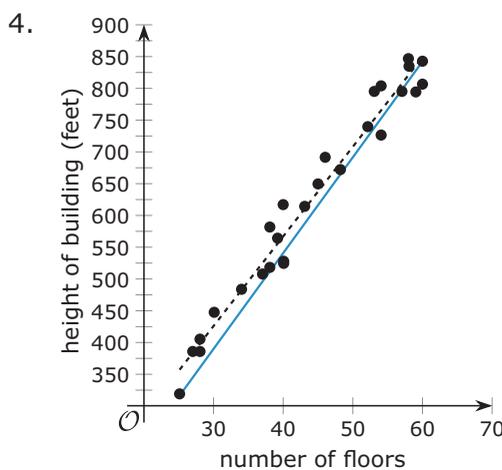
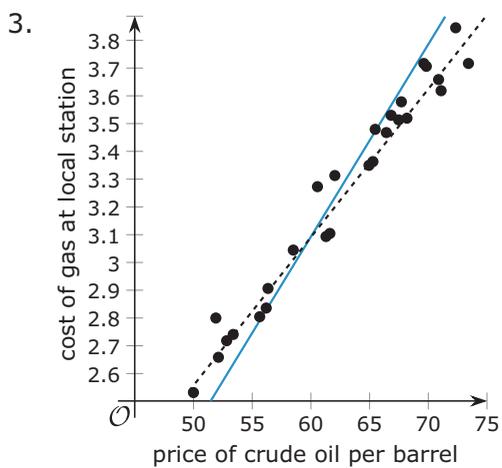
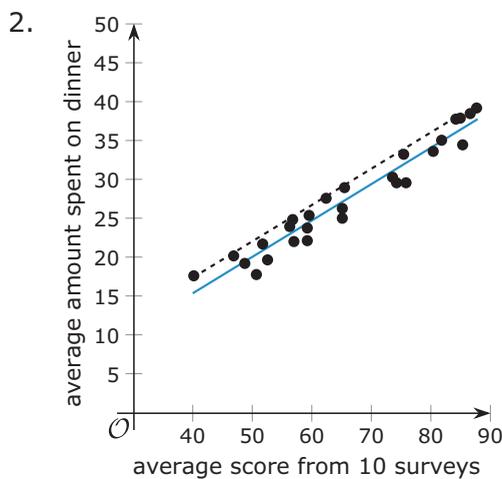
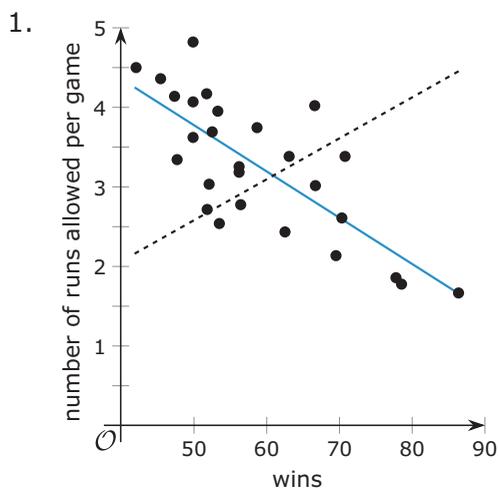


Unit 3, Lesson 5: Fitting Lines



Selecting the Best Line

Which of the lines is the best fit for the data in each scatter plot? Explain your reasoning.





Card Sort: Data Patterns

Your teacher will give you a set of cards that show scatter plots.

1. Arrange all the cards in three different ways. Ensure that you and your partner agree on the arrangement before moving on to the next one. Sort all the cards in order from:
 - A. best to worst for representing with a linear model
 - B. least to greatest slope of a linear model that fits the data well
 - C. least to greatest vertical intercept of a linear model that fits the data well
2. For each card, write a sentence that describes how y changes as x increases and whether the linear model is a good fit for the data or not.



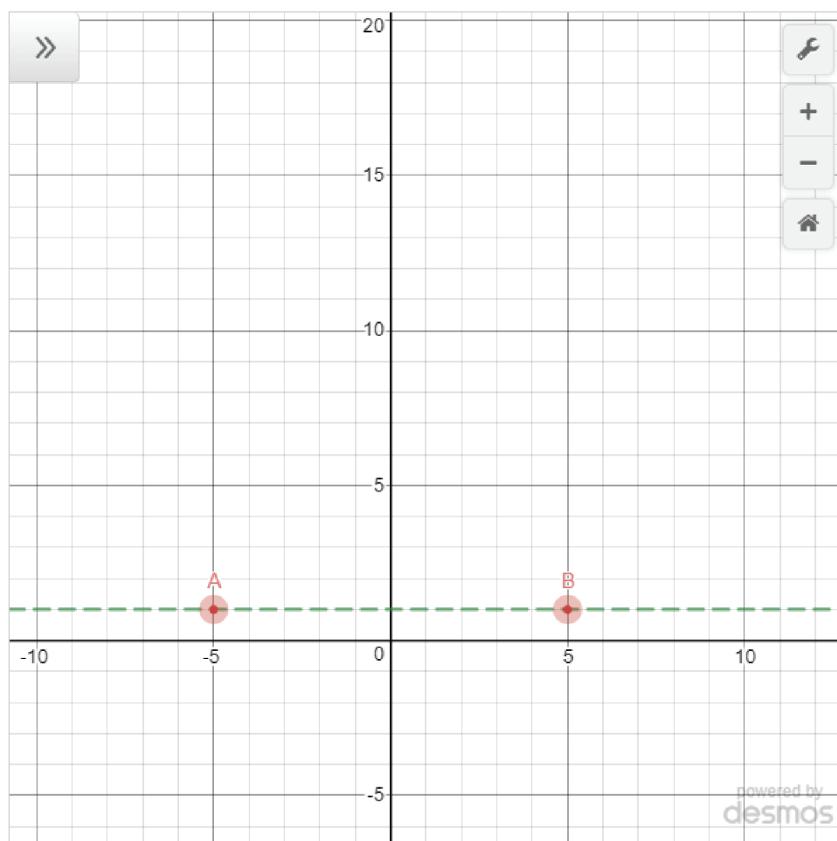
Fitting Lines with Technology



The weight of ice cream sold in a day at a small store in pounds (x) and the average temperature outside during the day in degrees Celsius (y) are recorded in the table.

x	y
20	6
18	4.5
21	6.5
17	3.5
21.5	7.5
19.5	6.5
21	7
18	5

This activity requires the use of an applet, so please make your way over to the digital platform to find the link.



1. For this data, create a scatter plot and a line that fits the data well.
2. Use technology to compute the best fit line. Round any numbers to 2 decimal places.
3. What are the values for the slope and y -intercept for the best fit line? What do these values mean in this situation?

4. Use the best fit line to predict the y value when x is 10. Is this a good estimate for the data? Explain your reasoning.

5. Your teacher will assign a table of data from the previous activity. Following your teacher's directions, use technology and the table of data to create a scatter plot that also shows the line of best fit, and then interpret the slope and y -intercept.

Tables for last question:

- A. (card A in the previous activity)
- B. (card B in the previous activity)
- C. (card C in the previous activity)
- D. (card E in the previous activity)
- E. (card F in the previous activity)

x	y	x	y	x	y	x	y	x	y
1	2	1	11.86	1	6.11	1	13.9	1	6.5
2.2	4	2.2	11.332	2.2	7.142	2.2	11.5	2.2	8.5
3.3	5	3.3	10.848	3.3	8.088	3.3	9.3	3.3	9.5
3.3	4.5	3.4	10.741	3.5	8.19	3.5	9.2	3.3	9
3.6	6	3.6	10.716	3.6	8.346	3.6	8.7	3.6	10.5
3.8	6.5	3.8	10.628	3.8	2.92	3.8	8.3	3.8	11
3.9	5.7	3.9	10.584	3.9	8.604	3.9	8.1	3.9	10.2
4	7	4	10.54	4	8.69	4	7.9	4	11.5
4.4	6.5	4.4	10.364	4.4	9.034	4.4	7.1	4.4	11
4.5	7	4.5	10.32	4.5	9.12	4.5	6.9	4.5	11.5
4.7	7	4.7	10.232	4.7	9.292	4.7	6.5	4.7	11.5
4.8	6	4.8	10.188	4.8	13.6	4.8	6.3	4.8	10.5
4.9	8.7	4.9	10.144	4.9	9.464	4.9	6.1	4.9	13.2
5	7	5	10.1	5	9.55	5	5.9	5	11.5
5.1	7.7	5.1	10.056	5.1	9.636	5.1	5.7	5.1	12.2
5.2	6.7	5.2	10.5	5.2	9.722	5.2	5.5	5.2	11.2
5.5	8	5.5	9.88	5.5	9.98	5.5	4.9	5.5	12.5
5.5	8.5	5.7	9.753	5.8	10.32	5.8	4.3	5.5	13
6	9.5	6	9.66	6	10.41	6	3.9	6	14
6.6	8.6	6.6	9.396	6.6	10.926	6.6	1.3	6.6	13.1
7	9	7	9.22	7	11.27	7	1.9	7	13.5
7.7	10.313	7.7	8.912	7.7	11.872	7.7	0.5	7.7	14.813



Practice Problems

This activity requires the use of an applet, so please make your way over to the digital platform to find the link.

1. Technology required.

x	y
83	102
87	115
91	107
93	122
97	125
97	127
101	120
104	127

- A. Use graphing technology to create a scatter plot and find the best fit line.

- B. What does the best fit line estimate for the y value when x is 100?

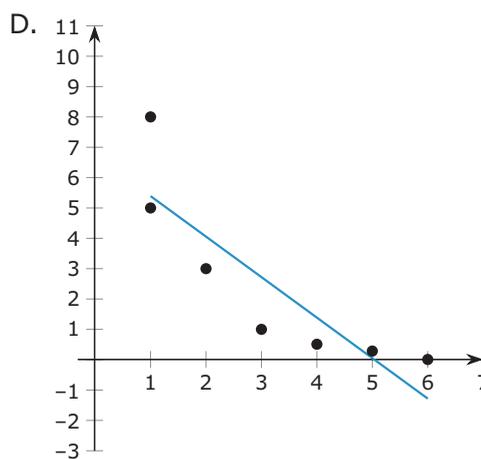
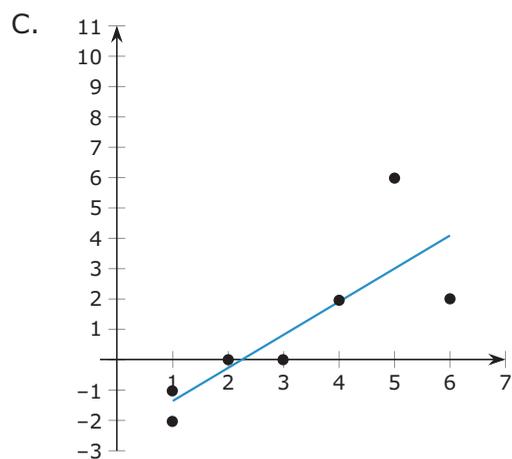
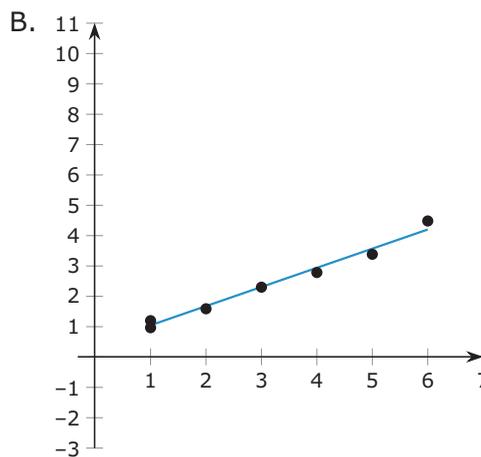
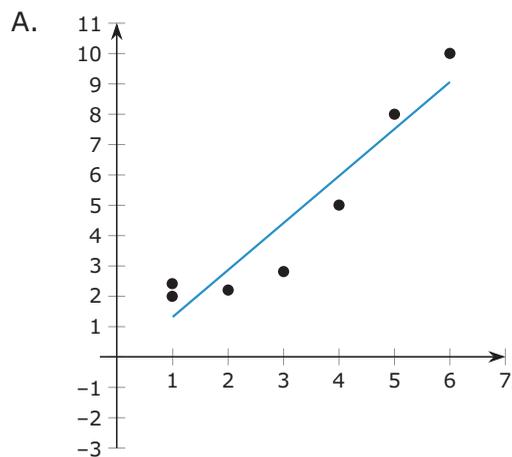
This activity requires the use of an applet, so please make your way over to the digital platform to find the link.

2. Technology required.

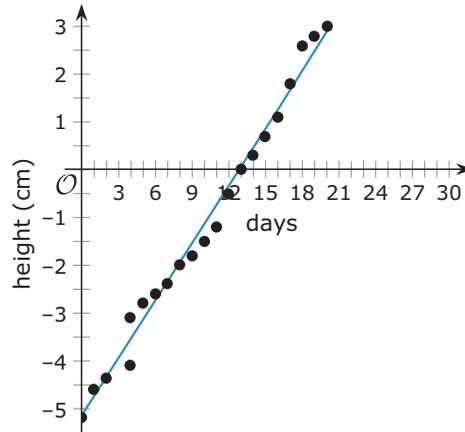
x	y
2.3	6.2
2.8	5.7
3.1	4.7
3	3.2
3.5	3
3.8	2.8

- A. What is the equation of the line of best fit? Round numbers to 2 decimal places.
- B. What does the equation estimate for y when x is 2.3? Round to 3 decimal places.
- C. How does the estimated value compare to the actual value from the table when x is 2.3?
- D. How does the estimated value compare to the actual value from the table when x is 3?

3. Which of these scatter plots are best fit by the shown linear model?



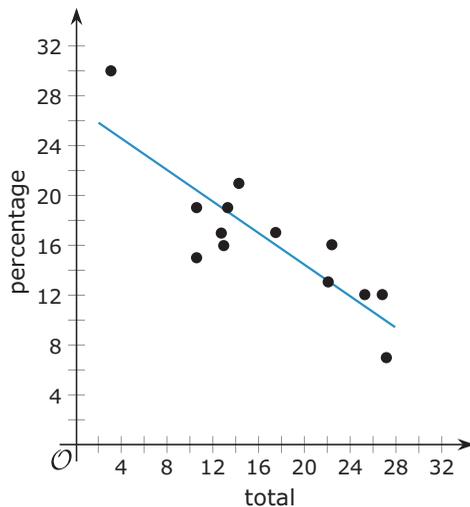
4. A seed is planted in a glass pot and its height is measured in centimeters every day.



The best fit line is given by the equation $y = 0.404x - 5.18$, where y represents the height of the plant above ground level, and x represents the number of days since it was planted.

- A. What is the slope of the best fit line? What does the slope of the line mean in this situation? Is it reasonable?
- B. What is the y -intercept of the best fit line? What does the y -intercept of the line mean in this situation? Is it reasonable?

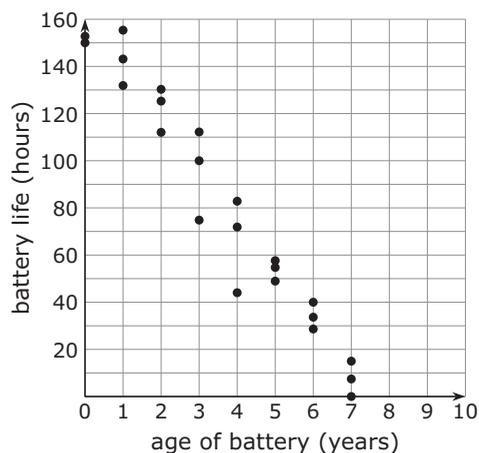
5. At a restaurant, the total bill and the percentage of the bill left as a tip are represented in the scatter plot.



The best fit line is represented by the equation $y = -0.632x + 27.1$, where x represents the total bill in dollars, and y represents the percentage of the bill left as a tip.

- A. What does the best fit line estimate for the percentage of the bill left as a tip when the bill is \$15? Is this reasonable?
- B. What does the best fit line predict for the percentage of the bill left as a tip when the bill is \$50? Is this reasonable?

6. A recent study investigated the amount of battery life remaining in alkaline batteries of different ages. The scatter-plot shows this relationship between the different alkaline batteries tested.

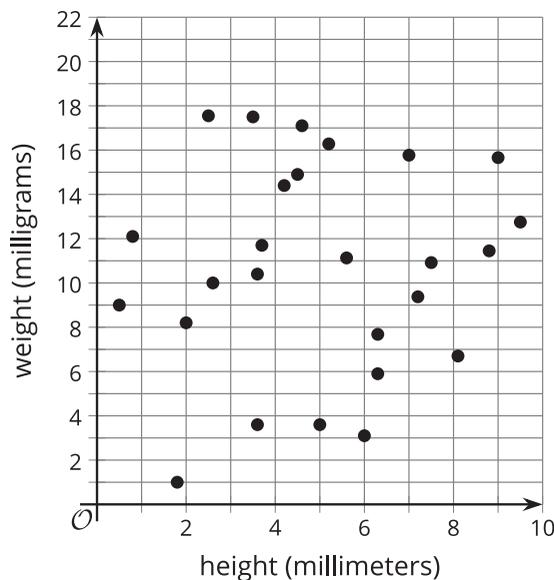


The scatter plot includes a point at (7, 15). Describe the meaning of this point in this situation.

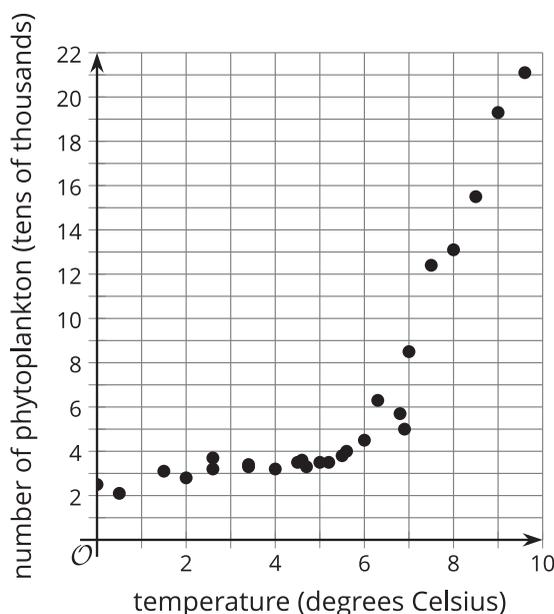
Lesson 5: Fitting Lines

Cool Down: Fresh Air

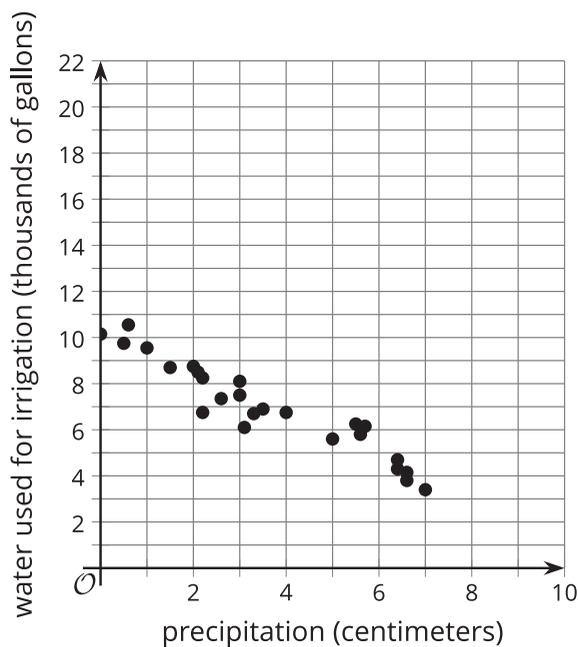
1. Which of these scatter plots shows data that would best be modeled with a linear function? Explain your reasoning.



a.



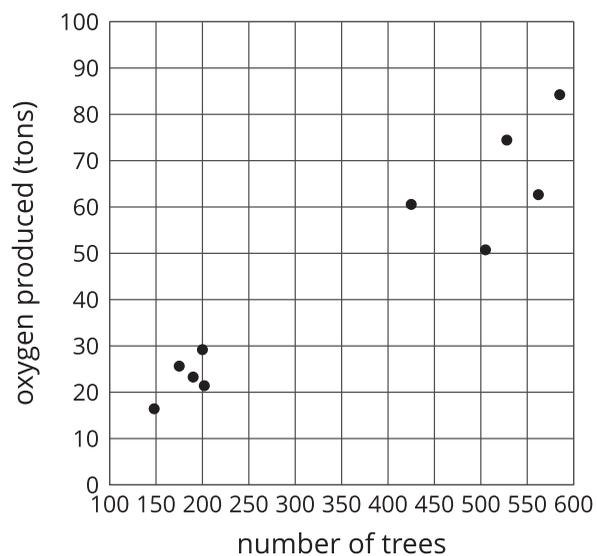
b.



c.

2. Which of the lines is most likely the line of best fit for the data provided?

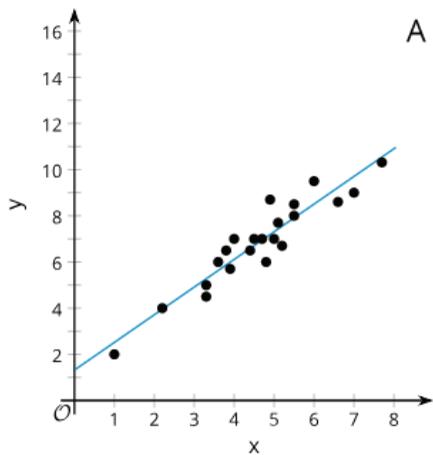
number of trees in a forest	tons of oxygen produced by the forest
148	16.43
175	25.64
190	23.28
200	29.2
202	21.41
425	60.56
505	50.75
528	74.45
562	62.66
585	84.24



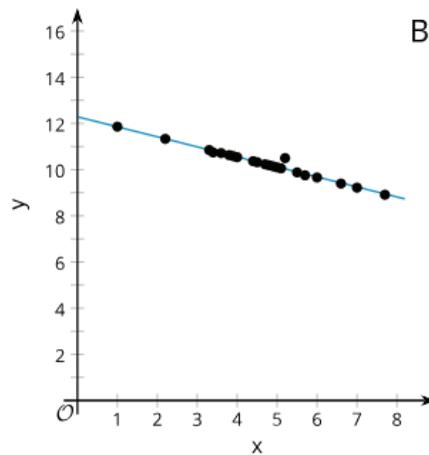
- a. $y = -0.51x + 225.12$
- b. $y = 0.34x - 34.05$
- c. $y = 0.13x - 0.19$
- d. $y = 0.98x - 21.13$

Alg1.3.5.2 Card Sort: Data Patterns.

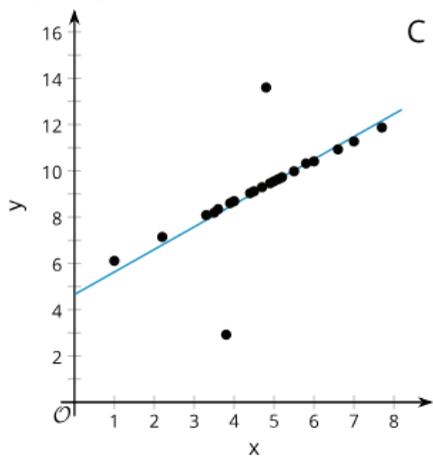
Card Sort: Data Patterns



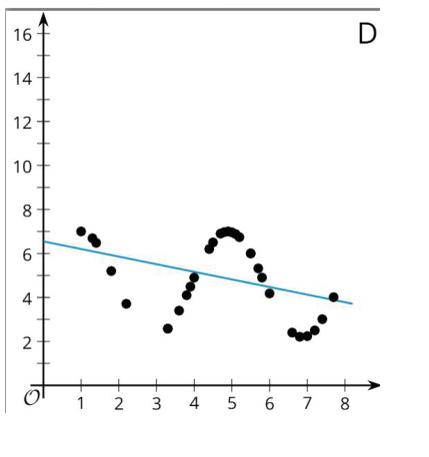
Card Sort: Data Patterns



Card Sort: Data Patterns

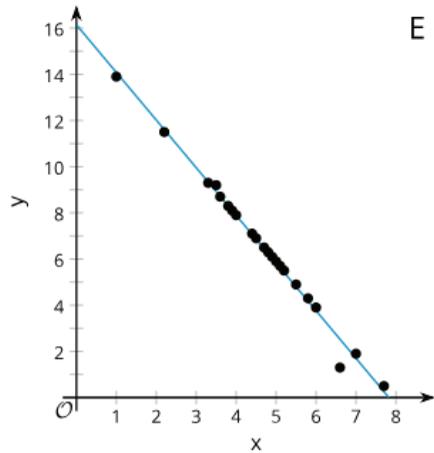


Card Sort: Data Patterns



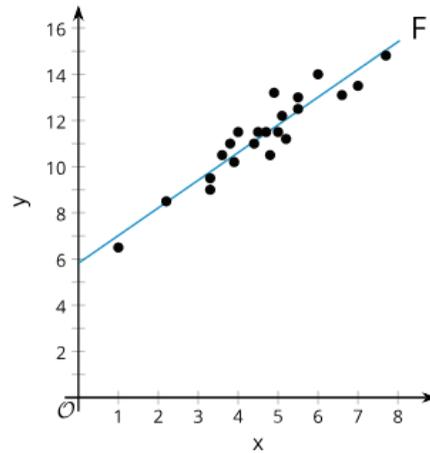
Alg1.3.5.2 Card Sort: Data Patterns.

Card Sort: Data Patterns



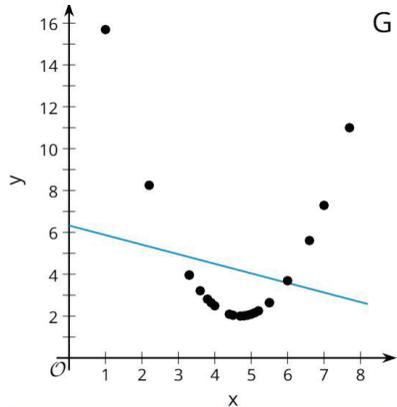
E

Card Sort: Data Patterns



F

Card Sort: Data Patterns



G

Goodness of Fit

- Let's explore lines and their goodness of fit for data

What's the Rate?

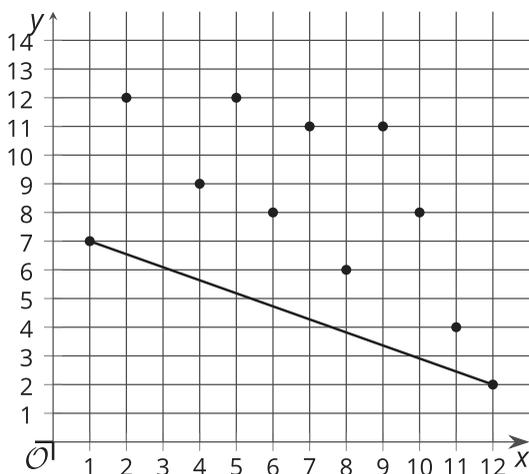


Each situation can be modeled using a linear equation. Describe the rate of change for each situation.

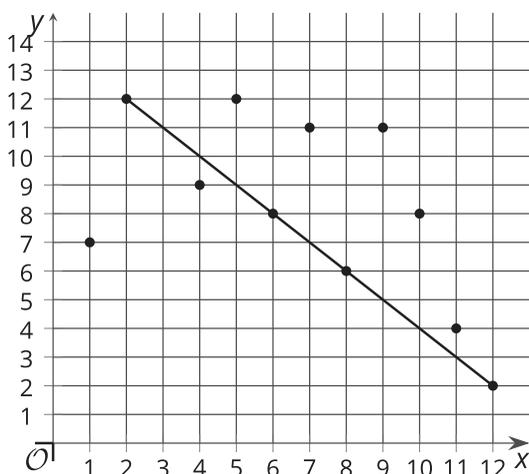
1. Andre started his no-interest savings account with \$1,000. He makes the same deposit each week, and there is \$1,600 in the account after 6 weeks.
2. Kiran starts with \$748 in his checking account. After 4 weeks of spending the same amount each week, he has \$716 left.

Goodness of Fit

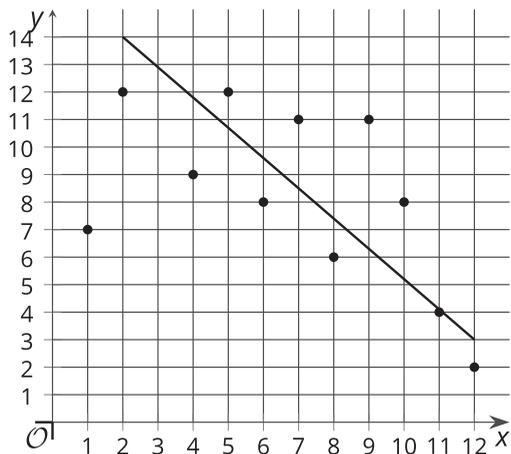
Here are 3 copies of the same scatter plot. Each student tries to draw a line that models the data well.



Noah says his line fits the data well because the line connects the leftmost point to the rightmost point.



Andre says his line fits the data well because it passes directly through as many points as possible.

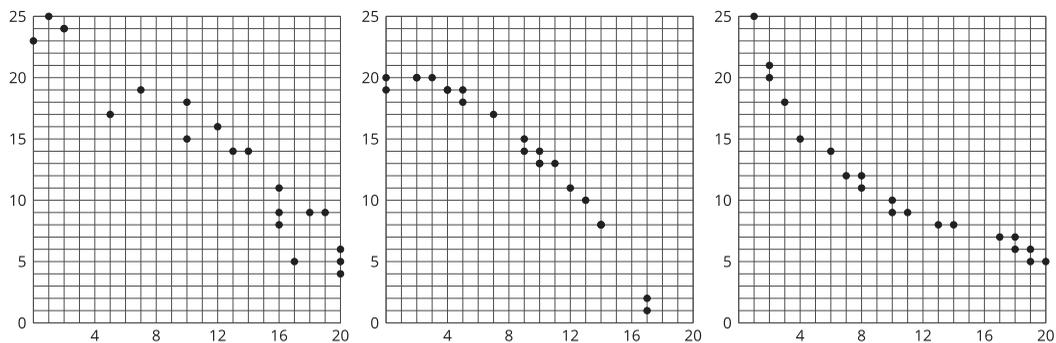


Lin says her line fits the data well because the points are somewhat evenly arranged around the line with about half the points above the line and half the points below the line.

Do you agree with any of these students? Explain your reasoning.

What Fits?

1. Look at the scatter plots, and determine which one is best modeled by a linear model.



2. Draw a linear model that fits the data well on the appropriate scatter plot. Compare your line with a partner's. If your lines are different, determine which line is the better fit line.

Lesson 5: Goodness of Fit

These materials, when encountered before Algebra 1, Lesson 5: Fitting Lines, support success in that lesson.

Goals

- Understand what makes a linear model a better or worse fit for data

Lesson Narrative

The mathematical purpose of this lesson is for students to understand what makes a linear model a better or worse fit for data displayed using a scatter plot. In the associated Algebra 1 lesson, students create lines of best fit and interpret the y -intercept of a linear model in terms of the situation. Students construct viable arguments (MP3) when they explain the goodness of fit of lines created using various methods.

Student Learning Goals

- Let's explore lines and their goodness of fit for data

5.1 What's the Rate?

Warm Up: 5 minutes

The goal of this activity is for students to determine the rate of change for situations. This activity prepares students to interpret the slope of linear models in the associated Algebra 1 lesson.

Student Task Statement



Each situation can be modeled using a linear equation. Describe the rate of change for each situation.

1. Andre started his no-interest savings account with \$1,000. He makes the same deposit each week, and there is \$1,600 in the account after 6 weeks.
2. Kiran starts with \$748 in his checking account. After 4 weeks of spending the same amount each week, he has \$716 left.

Student Response

1. Andre deposited \$100 each week.
2. Kiran spent \$8 each week.

Activity Synthesis

The goal of this activity is for students to practice finding the rate of change in a given scenario. Discuss how students can find the rate of change. Here are sample questions to promote class discussion:

- “How do you determine the rate of change for Andre’s savings account?” (First, I think about the difference between how much money he had in the beginning and the end of the situation. Once I determine the difference, then I use the 6 weeks to divide to get the rate of change.)

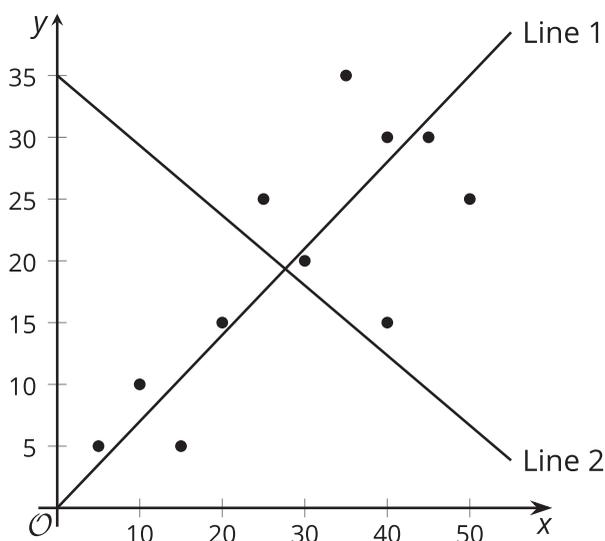
- “How do you determine the rate of change for Kiran’s checking account?” (First, I think about the difference between how much money he had in the beginning and the end of the situation. Once I determine the difference, I set up a proportion to describe what he spent over 4 weeks and to calculate how much that meant he spent each week.)
- “How does rate of change relate to a linear model?” (The rate of change relates to a linear model because linear models show a constant increase, decrease, or no change in the variables. The rate of change tells me how much a variable increases, decreases, or if it does not change at all. Similarly, in a linear model, the slope tells us how much the line increases, decreases, or if it does not change at all. The rate of change and the slope give us the information on how a variable or line changes, respectively.)

5.2 Goodness of Fit

20 minutes

The purpose of this activity is for students to think about how to determine if a linear model is a better or worse fit for data displayed in a scatter plot. This activity prepares students to create lines of best fit in the associated Algebra 1 lesson.

Launch

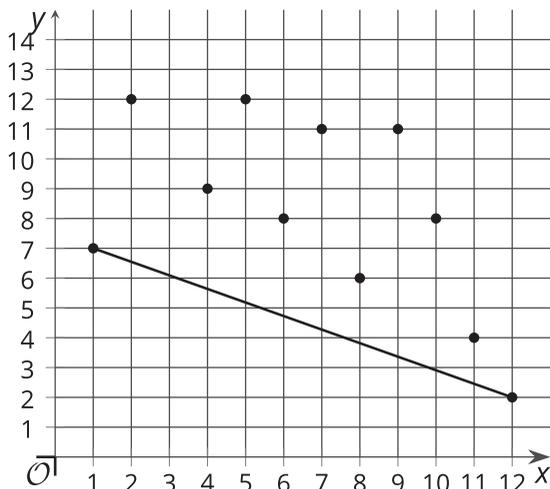


Display the scatter plot for all to see.

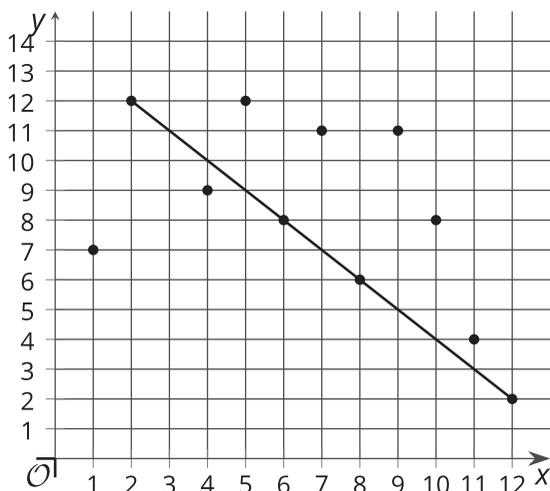
Ask students which line appears to fit the data best. Ask them to explain their reasoning. Listen for students who describe the trend formed by the data or uses the term *line of best fit*. Line 1 is a better fit to the data than line 2 because it follows the same trend as the one formed by the points. For students who have forgotten about fitting lines, this image can help them recall what makes a line a good fit for a data set.

Student Task Statement

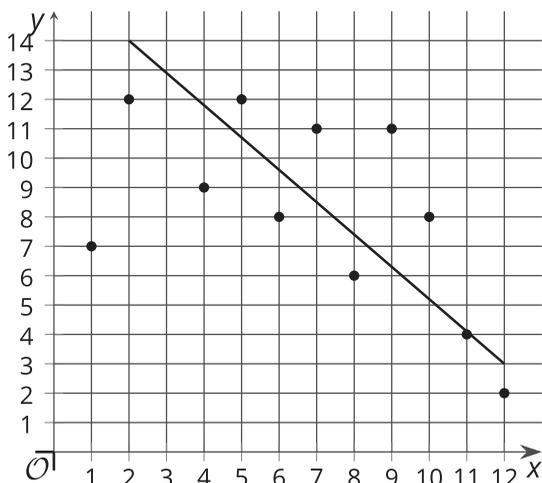
Here are 3 copies of the same scatter plot. Each student tries to draw a line that models the data well.



Noah says his line fits the data well because the line connects the leftmost point to the rightmost point.



Andre says his line fits the data well because it passes directly through as many points as possible.



Lin says her line fits the data well because the points are somewhat evenly arranged around the line with about half the points above the line and half the points below the line.

Do you agree with any of these students? Explain your reasoning.

Student Response

Sample Response: I agree with Lin because if one line is used as a model for a data set, then the line should represent all of the data points as much as possible. By ensuring that the points are evenly arranged around the line, the model is a neutral location and follows the same trend as the data set.

Activity Synthesis

The goal of this discussion is for students to think about what makes a linear model a good fit for a data set. Discuss what would make a linear model a good fit. Here are sample questions for discussion:

- “What do you think makes a line a good fit for a data set?” (A good fit line goes through the middle of the data and follows the trend formed by the points. A bad fit would not show the same trend as the one formed by the points or not go through the middle of the data.)
- “What do you think is an advantage of using a linear model to represent data?” (Using a linear model to represent data allows you to get an overall view of the trend of the data. You can tell if the data are increasing or decreasing overall, even if they do not do so with each new data point. You can also predict or estimate additional information because the linear model helps to establish a quantifiable pattern in the data.)

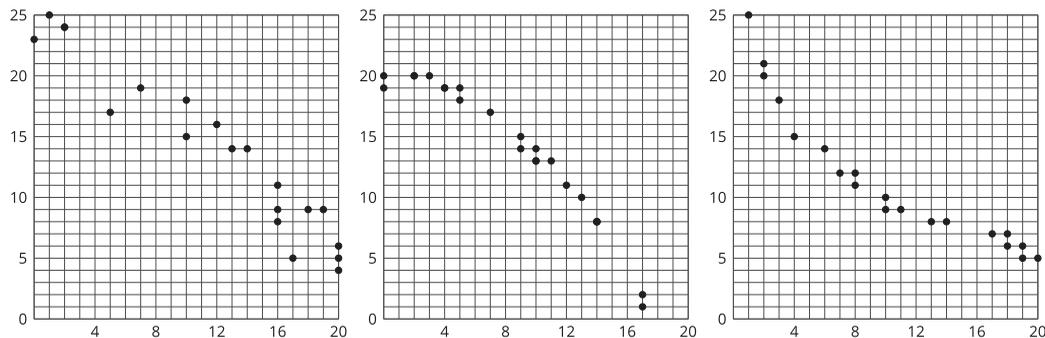
5.3 What Fits?

15 minutes

The purpose of this activity is for students to learn how to create a linear model that is a good fit to data. Students first determine which scatter plots are best represented by a linear model. Then, students create their own linear model that is fit to the data, and compare their lines with a partner’s.

Student Task Statement

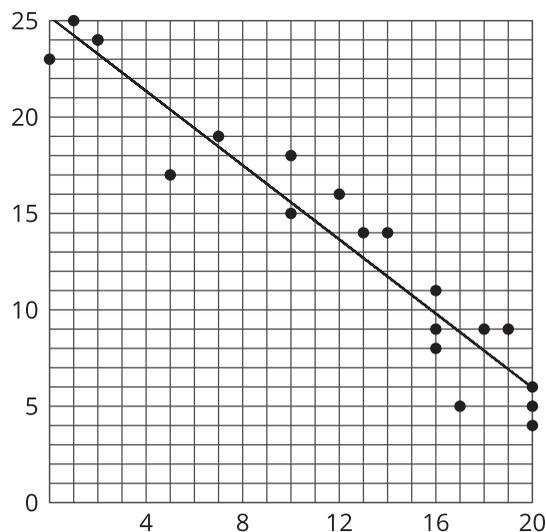
1. Look at the scatter plots, and determine which one is best modeled by a linear model.



2. Draw a linear model that fits the data well on the appropriate scatter plot. Compare your line with a partner's. If your lines are different, determine which line is the better fit line.

Student Response

1. Graph A is the scatter plot that is best modeled with a linear model.



2.

Activity Synthesis

Discuss how students arrived to their answers. Here are sample questions for discussion:

- “How do you determine if a scatter plot is best modeled by a linear model?” (The scatter plot with points that are closest to forming the shape of a straight line is the scatter plot that is best modeled by a linear model.)
- “For groups that had different lines, how did you determine whose line was more accurate?” (We looked closely at each of the lines and decided that the better fit line was the one that had the points more evenly arranged around it.)



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